Unification of the Greek vertical datum through a deterministic adjustment of tide gauge, marine geoid and sea surface topography data

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Abstract. *Countries like Greece with extensive coastlines and a large number of islands usually suffer from the absence of a common, for the entire country, unified vertical reference system. In Greece, no effort has been made until today for the unification of the country's vertical datum especially between mainland and the insular part. The main source for the vertical datum offsets is the sea surface topography, especially the stationary part, which is in effect the difference between the mean sea level realized by the tide-gauge stations at the islands and the one at Piraeus that indicates the origin of the country's vertical datum. The present work focuses on the utilization of available tide gauge and spirit levelling data with computed marine geoid and sea surface topography models, towards the determination of a common corrector surface for continental and insular Greece in order to unify the country's vertical datum. The aforementioned corrector surface provides correction values to be applied to local tide gauge mean sea level revords, so that the local zero level will coincide with that at the origin of the vertical system. The concept is based on a common adjustment of the available data in a parametric scheme imposing a condition concerning the value of the corrector model at the existing vertical origin of the country. The necessary observation equations are outlined together with the theoretical concepts of the data combination scheme. Various reference surfaces are investigated and validated against each other and in terms of the prediction error they provide. The results of this work successfully manage to provide correction values for the entire country, so that local heights tied to a local tide gauge station can be referred to the initial point of the country's vertical datum.*

Keywords: *Height unification, tide gauge, sea surface topography, combination. vertical reference system.*

1. Introduction

The importance of orthometric heights, i.e., heights above mean sea level, to all geodetic, surveying, and engineering applications is significant, since they (among others) (a) determine land topography, (b) provide a conceptual framework for citizens to comprehend the altitude of their property, (c) are used by surveyors and other engineers in field operations such as cadastral, road construction, environmental, water management and other works, and (d) are used in geodetic practice as displacements along the local vertical to a reference surface (the geoid) to define with the horizontal coordinates the geo-location of a point. Featherstone and Kuhn (2006) give a detailed discussion on the various height systems, the need for a unified network and their importance for surveying applications, which, even though it is directed to the Australian case does not lose its generality. In most cases orthometric heights are determined nationwide by conventional spirit levelling accompanied by gravity measurements along dedicated traverses, starting from a selected benchmark (BM) that serves as the origin or zero point of the country's vertical reference system. The orthometric heights of all established benchmarks are then determined, through a dedicated adjustment of the entire vertical network, as height differences with respect to (w.r.t.) the origin of the country's network. It has been customary for the initial point to coincide with a tide gauge (TG) station, so that through the record of data of the latter the local mean sea level (MSL) would be determined and a local geoid would be established as the zero level surface W_0, w.r.t. which all orthometric heights would be referred and measured thereafter. Therefore, the orthometric height of a point P on the Earth's surface is determined through the potential difference ΔW_P between the Earth's gravity potential W_P at that point and the potential of the vertical system's zero level W_0 (Heiskanen and Moritz, 1967). In a practical sense, the orthometric height differences measured by traditional spirit levelling and combined with gravimetric measurements correspond to the potential difference ΔW_P.

This approach has some limitations in countries like Greece, with the main problem being the existence of hundreds of islands, where no hydrostatic levelling has been applied in order to connect the orthometric heights of the BMs at the islands with the origin of the country's vertical system. In such cases, the orthometric heights at an island refer to the MSL of a local TG station, i.e., to a different zero level W_0^{local}, so that they are not tied to the country's vertical network. Moreover, the vertical network of Greece has not been uniformly adjusted in a single step, where all levelling traverses are taken into account. As a result of the aforementioned problems, Greece does not have a unified vertical network with all orthometric heights referring to the same level of the supposed origin, which is located in the TG station at the Piraeus harbour.

Until recently, the solution to such a problem, as far as the mainland stations are concerned, would be only through a common adjustment of the levelling traverses already acquired, so that a unified adjustment of the entire vertical network would be achieved. Given that these data are mainly historical, have been collected over a long period of time, with different instruments and present varying accuracy, and the fact that their status remains confidential by the Hellenic Military Geographic Service (HMGS), a unified adjustment of the entire network cannot be performed. As far as the BMs at the islands are concerned, the problem of their connection to the zero level of the country presents a vital, and until recently irresolvable difficulty, i.e., the need to perform hydrostatic levelling in order to transfer heights across marine areas and tie some BMs of the island to the zero level of the country's vertical datum. Then, the rest of the BMs available at the island can be connected with conventional spirit levelling to the zero level of the network. Such a procedure for countries like Greece with more than 200 inhabited islands would be time-consuming, costly and the final accuracy achieved would be largely doubtful in order to justify such a massive operation. On the other hand, the advance in space geodetic techniques and the availability of high-resolution and high-accuracy data for marine areas from altimetry offer some new opportunities for the unification of the Greek vertical network and the successful connection of the levelling network at the islands with the zero level at Piraeus. The formulation of the problem, as it will be presented in the next section, is based on the fact that the deviation between a local zero level at a TG station of an island w.r.t. the zero level of the TG at Piraeus is simply the difference between the MSL recorded in the two stations, which coincides with the quasi-stationary sea surface topography (QSST). The same holds for mainland stations too, which are connected through levelling to the zero level of the country. Therefore, if a high-accuracy and high-resolution QSST model is available, along with QSST values for TG stations which are connected to the country's zero level, then a combined adjustment can be performed, in order to determine a fitted QSST model, which will provide "correction" values for the rest of the country.

The concept of using satellite and terrestrial gravity field related data in order to unify vertical datums at nationwide, regional and global scales is well established and with the recent GOCE mission (ESA, 1999) becomes even more apparent. With the early exact repeat mission data from GEOSAT, Burša et al. (1992) have determined the geopotential scale factor, while using a 6 year record (1993-1999) from the TOPEX/Poseidon satellite Burša et al. (1999) have provided an estimate of W_0 and proposed its adaptation for the determination and realization of world height system (WHS). The establishment of a W_0 value and its adaptation for the realization of WHS can then serve in order to unify local vertical datums (LVDs) into a global one by determining (vertical) shifts for each one w.r.t. the WHS. This has been used by Burša et al. (2001) in order to find the geopotential differences between LVDs and unify them into a WHS, and by Burša et al. (2004) to determine a global vertical reference frame through the unification of the North American, Australian, French and Brazilian height datums. Finally, Burša et al. (2002) practically demonstrated the realization of a WHS by determining geopotential values at

TG stations and the geopotential differences between the LVDs and the global one. Similar attempts at a regional level has been presented, amongst others, by Featherstone (2002) where an attempt for the unification of the Australian Height Datum has been presented, Amos and Featherstone (2009) who presented an iterative scheme of quasigeoid computation for the unification of New Zealand's vertical datums and Merry (2003) for the unification of the African vertical datums within the African geoid project. At the European level, there has been a significant amount of work performed towards the determination of a European Vertical Reference System (EVRS) by combining levelling data from many European countries (Ihde et al., 2002). The absence of Greek levelling data from such an effort (see the current list of network data in EVRS, 2010) and the practical exclusion of the country from such a system give good evidence on the necessity of the present study. This is apparent since not only the Greek levelling network is not connect to the EVRS, but the network itself is not consistent and unified for the entire country. For a more intuitive look at the geoidal geopotential, vertical datums and the various types of heights within datum unification Colombo (1980), Jekeli (2000), Kearsley et al. (1993), Kearsley (2004), Rummel and Teunissen (1998) should be consulted.

2. Observation equations for datum homogenization and data

In order to define the observation equations and layout the problem of datum homogenization for Greece, lets assume that we have the ideal case scenario presented in Figure 1. Within this ideal case, a TG station is available where collocated observations are performed, i.e., the local MSL H_{MSL} and instantaneous sea level (ISL) H_{ISL} are determined by the TG measurements, spirit levelling has been performed in order to connect the TG station to the zero level of the vertical datum through the orthometric height difference $\Delta H_{BM\text{-}TG}$ w.r.t. to an available BM whose orthometric height H_{BM} is known, and the ellipsoidal heights of both the TG station h_{TG} and levelling BM h_{BM} are known through GPS measurements. Moreover, satellite altimetry data are available so that the sea surface height SSH is known, a gravimetric geoid model is available for the region so that the geoid height N of the TG station is available and the combination of geodetic, altimetric and oceanographic data provide an independent, from the TG station measurements, estimation of the QSST ς_c and the time-varying SST ς_t. With this abundance of information, one can combine the available heights and form the observation equations both for land and marine areas in order to homogenize the vertical datum, as follows:

$$h_{TG} - H_{TG} - N = 0 \quad \textit{on land}, \tag{1}$$

$$SSH - N - \varsigma_c = 0 \quad \textit{at sea}. \tag{2}$$

Eq. (1). describes the situation on land, where we can combine the ellipsoidal, orthometric and geoid heights in order to form the observation equation while, in complete analogy, the SSH, sea surface topography and geoid height are combined at sea. Such data can be used in order to update/improve the connection of the TG station to the zero level of the country and to transfer its connection to the zero level of another station by performing *levelling* at sea since the triplet of *SSH*, ς_c and N are available for the marine areas (Fotopoulos, 2004). Note that in Eq. (2), the role of the time-varying SST denoted by ς_t in Figure 1 is not omitted, but it is considered that ς_t is contained in the noise of the observations and treated by the parametric models used to adjust the differences between the QSST, the sea surface and geoid heights (see Eq. (3) that follows and the discussion in Section 2).

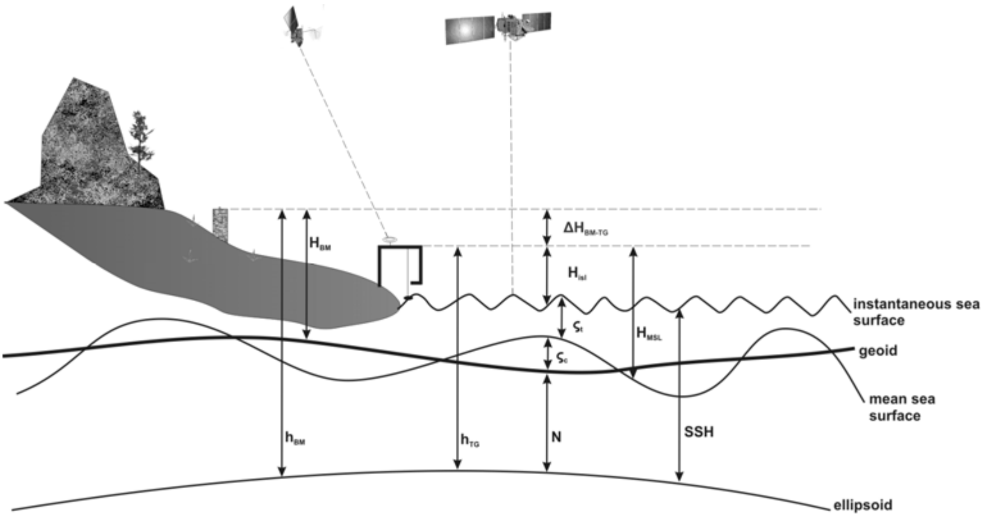

Figure 1: Combination of TG, GPS, levelling and satellite altimetry data for height unification.

Such an abundance of information and data cannot be found for the Greek VRS (Vertical Reference System), since GPS measurements at the TG stations are not available and in many cases the levelling connection with the zero level of the country is missing. Figure 2 presents the so-called *good scenario* for the Greek vertical datum homogenization, i.e., the case that a connection through levelling between the local TG and the zero level of the country is available. Figure 2 presents such a case, where a) the local MSL H_{MSL} and ISL H_{ISL} are determined by the TG measurements and b) there is a levelling connection $\Delta H_{TG\text{-}BM}$ between the TG station and a BM, whose orthometric height H_{BM} is known w.r.t. to the TG station of Piraeus. Note that the zero level determined by the local TG station W_0^{local} differs from the zero level W_0 of the country as determined by the TG station of Piraeus. Therefore, the deviation between these two equipotential surfaces is simply the difference between the MSL as determined by the two TGs and equals to the QSST, so that we can write:

$$\Delta \mathrm{H} = \mathrm{H}_{\mathrm{BM}} - \Delta \mathrm{H}_{TG-BM} - H_{MSL} = \varsigma_c^{MSL} \neq 0\,. \tag{3}$$

Eq. (3) would be equal to zero only in case the two MSLs determined by the local TG and that at Piraeus would have the same value, i.e., if their measurements realized the same zero level. It should be noted that we denote with $\varsigma_c^{\mathrm{MSL}}$ the QSST as determined by the TG measurements and data in order to distinguish it from ς_c which denotes the QSST as determined form an available model.

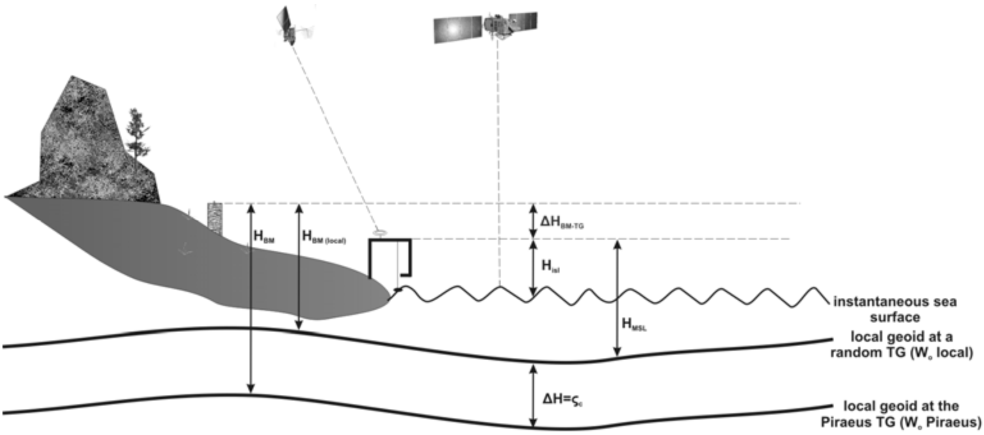

Figure 2: Height unification for the Greek case: layout of TG, sea surface topography and levelling data.

It has already been mentioned, that this is the *good scenario* for the Greek VRS, while the usual case, especially for the islands is depicted in Figure 2 as well. In the *usual scenario* the local MSL H_{MSL} and ISL H_{ISL} are determined by the TG measurements and there is a levelling connection $\Delta H_{TG\text{-}BM}$ between the TG station and a BM, whose orthometric height $H_{BM\ (local)}$ is not known w.r.t. to the TG station of Piraeus but only w.r.t. to the local TG. Note that for such a station, and for BMs that are connected through levelling to it, the zero level they refer to is that determined by the local TG station W_0^{local} and not the zero level of the country. Therefore, if a QSST model was available for these stations, in order to estimate their difference ΔH in Eq. 3 w.r.t. the zero level of the country, then it would be possible to refer their heights and those of all other BMs that are determined from them to the origin of the Greek VRS. This forms the basis for the unification of the Greek VRS presented in this work, where a QSST model (ς_c) for the entire Greek territory is adjusted in order to fit to the QSST as determined by the TG measurements (ς_c^{MSL}) for those stations that are connected to the zero level of the Greek VRS. Through this combined adjustment, a new improved QSST model can be determined, so that it will provide *correction* values (ΔH), which when applied to local TG stations will refer they W_0^{local} to the zero level W_0 of Piraeus. A crucial point for the aforementioned to hold is that all MSL observations at the TG stations (both the local ones and that at Piraeus) are free from any time-varying effects and from the influence of tides. Such effects are assumed to have been taken into account and removed from the observations, together with the effect of time-varying SST, so that the only part of the sea surface topography that remains is the quasi-stationary one. It is acknowledged that Greece is a seismically active country, therefore, some vertical motion may occur and can therefore distort the vertical datum of the country. Such vertical displacements that would result in datum deviations and can be contained in the data used in the presented study, are considered to be absorbed by the parametric models used.

Within such a frame, we can form the general model of the problem's observation equations as

$$\ell_i = \left(\mathrm{H}_{BM}^i - \Delta\mathrm{H}_{TG-BM}^i - H_{MSL}^i\right) - \varsigma_c^i = \varsigma_c^{MSL\,i} - \varsigma_c^i = \mathbf{a}_i^T \mathbf{x}_i + v_i\,, \tag{4}$$

where, ℓ_i denotes the observation for the TG station *i*, the elements $\mathbf{a}_i^T$ are those of the design matrix $\boldsymbol{A}$ and the unknowns x_i depend on the parametric model chosen to describe the differences between the triplet of heights and v_i are the observation errors. In Eq. (4), $\varsigma_c^{MSL\,i}$ denotes the QSST as determined by the *i* TG station measurements whereas ς_c^i denotes the QSST from the available model. For the parametric model $\mathbf{a}_i^T \mathbf{x}$ to be used, there is a range of choices from the well-known three-, four- and five-parameter similarity transformation ones, polynomial models of various degrees, trigonometric functions based on Fourier analysis, etc. A nice review of the various selections available and an analysis of their physical meaning are given in Fotopoulos (2003). For the unification of the Greek vertical reference system, the three-, four and five-parameter similarity transformation models have been used along with the 1st and 2nd order polynomial ones and a QSST dependant parametric model which constrains the relation between the TG-derived and the altimetric-gravimetric derived sea surface topography models. The parametric models used are given in in Eqs. (5)-(9) respectively (Fotopoulos, 2003; Heiskanen and Moritz, 1967; Kotsakis and Sideris, 1999)

$$\mathbf{a}_i^T \mathbf{x} = x_0 + x_1 \cos\phi_i \cos\lambda_i + x_2 \cos\phi_i \sin\lambda_i\,, \tag{5}$$

$$\mathbf{a}_i^T \mathbf{x} = x_0 + x_1 \cos\phi_i \cos\lambda_i + x_2 \cos\phi_i \sin\lambda_i + x_3 \sin\phi_i\,, \tag{6}$$

$$\mathbf{a}_i^T \mathbf{x} = x_0 + x_1 \cos\phi_i \cos\lambda_i + x_2 \cos\phi_i \sin\lambda_i + x_3 \sin\phi_i + x_4 \sin^2\phi_i\,, \tag{7}$$

$$\mathbf{a}_i^T \mathbf{x} = \sum_{m=0}^{M} \sum_{n=0}^{N} x_q (\phi_i - \phi_0)^n (\lambda_i - \lambda_0)^m \cos^m \phi_i\,. \tag{8}$$

$$\mathbf{a}_i^T \mathbf{x} = \mu + \delta s_{\varsigma_c^i} \varsigma_c^i\,. \tag{9}$$

Eqs. (5)-(7) are simplified versions of the extended seven-parameter similarity transformation model. The 3-parameter model in Eq. (5) corresponding, apart from the bias term, to a north-south and an east-west component of an average spatial tilt between the two QSST models. The 4-parameter model in Eq. (6) corresponds geometrically to a 3D shift of the origin of the ς_c^i datum w.r.t. the LVD as realized by $\varsigma_c^{MSL\,i}$ and a bias term. The 5-parameter model in Eq. (7) introduces an additional term to the former and corresponds geometrically to a 3D shift of the origin, a bias and a scale change of the ς_c^i datum w.r.t. the LVD as realized by $\varsigma_c^{MSL\,i}$. Eq. (8) describes generally the well-known polynomial models, which have no physical meaning and they are used in many cases to provide a mathematical corrector surface for the fit between GPS/Levelling and gravimetric geoid heights, QSST models, etc. Finally, Eq. (9) is the aforementioned QSST dependant parametric model, which contains a single bias term μ to describe the constant offset between the global and the local vertical datums and a scale factor $\delta s_{\varsigma_c^i}$ to account for the different spatial scales between them. It should be noted that due to the few TG stations (eight) that were available for the datum homogenization, i.e., they were operational and connected to the zero level of the country with spirit levelling, higher-order polynomial models could not be tested in the adjustment in order to validate their performance. Moreover, the use of the polynomial models act more like a proof of concept to validate the performance of the three-, four- and five-parameter as well as the QSST-dependant one, since they have no physical meaning.

Since our goal is the homogenization of the Greek VRS w.r.t to the national zero level as determined by the TG station at Piraeus harbour, a constraint should be put in the system of observation equations and the adjustment. The meaning is that the correction that the determined model will provide at the TG location of Piraeus should be zero. This can be written as

$$\mathbf{a}_{PIRAEUS}^T \mathbf{x}_{PIRAEUS} = 0 \,. \tag{10}$$

which for, e.g., the five-parameter similarity transformation model and with φ_P and λ_P denoting the latitude and longitude of the Piraeus TG station, is

$$\begin{bmatrix} 0 & \cos\phi_P \cos\lambda_P & \cos\phi_P \sin\lambda_P & \sin\phi_P & \sin^2\phi_P \end{bmatrix} \mathbf{x} = 0 \,. \tag{11}$$

This constraint can be written in matrix notation as

$$\mathbf{Hx} = \mathbf{z} \,, \tag{12}$$

so that the final system of observation equations for the combined adjustment presented can now be written in the form:

$$\mathbf{b} = \mathbf{Ax} + \mathbf{v} \,, \quad \mathbf{P} = \left(\mathbf{Q}_{\varsigma_c^{MSL}} + \mathbf{Q}_{\varsigma_c} \right)^{-1} \,. \tag{13}$$

In Eq. (13), ***P*** denotes the observation weight matrix through the data covariance matrices $\mathbf{Q}_{\varsigma_c^{MSL}}$ and $\mathbf{Q}_{\varsigma_c}$. Based on Eqs. (12) and (13) we can now write the solution of the adjustment along with the prediction of the variance-covariance matrices of the unknowns and the errors as given in Eqs. (14)-(19).

$$\hat{\mathbf{x}}_o = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{b}\,, \tag{14}$$

$$\mathbf{S} = \mathbf{H}\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{H}^T\,, \tag{15}$$

$$\hat{\mathbf{x}} = \hat{\mathbf{x}}_o + \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{H}^T\mathbf{S}^{-1}\left(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}_o\right), \tag{16}$$

$$\hat{\mathbf{v}} = \mathbf{b} - \mathbf{A}\hat{\mathbf{x}}\,, \tag{17}$$

$$\mathbf{C}_{\hat{\mathbf{x}}} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1} - \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{H}^T\mathbf{S}^{-1}\mathbf{H}\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}, \tag{18}$$

$$\mathbf{C}_{\hat{\mathbf{v}}} = \mathbf{P}^{-1} - \mathbf{A}\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^T + \mathbf{A}\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{H}^T\mathbf{S}^{-1}\mathbf{H}\mathbf{N}^{-1}\mathbf{A}^T\,. \tag{19}$$

Then, the new adjusted QSST can be determined as

$$\varsigma_c^{adj} = \varsigma_c + \mathbf{a}_i^T\hat{\mathbf{x}}\,. \tag{20}$$

Some considerations that should be discussed refer to the selection of the QSST model that will be used in the adjustment. With the availability of satellite altimetry, marine and space gravimetry and oceanographic data increasing there is a large number of models that can be used. Most of them are based on an analysis of altimetric data (Engelis, 1985, 1987), in-situ oceanographic observations, while lately GRACE data have been incorporated as well (Knudsen, 1992; Knudsen and Tscherning, 2006; Pavlis et al., 1998; Rio and Hernandez, 2004; Vergos, 2002; Vergos and Tziavos, 2007). Moreover, some new proposals for the incorporation of GOCE data in the determination of the sea surface topography through a least squares collocation approach have been presented (Barzhaghi et al., 2009; Sansò et al. 2008). It is acknowledged that the sea surface topography models estimated through such combination techniques suffer in areas close to the coastline due to the increasing errors of altimetry as the satellite moves from marine to continental areas. Nevertheless, for the time being, this is the only feasible approach for the homogenization of the Greek VRS, so that the orthometric heights at the islands will refer to the zero level of the country. The alternate approach presented at the beginning of this section in Eqs. (1) and (2) is quite attractive, but needs GPS measurements at the TG stations, which are not available for Greece. In the present study, the QSST model used for the unification of the Greek VRS was estimated from a combination of satellite altimetry and marine gravity data and covers the entire Greek territory (Vergos and Tziavos, 2007; Vergos et al. 2005).

Given that Eq. (20) provides the adjusted QSST for the homogenization of the Greek VRS, there is clearly the need to address the goodness of fit it provides, estimate prediction errors and provide some measures for the quality check of the various parametric models. For all parametric models used, prediction errors have been estimated by leaving one of the available TG stations out of the adjustment and then estimating the sea surface topography value that the adjusted model provides and comparing it to the ς_c^{MSL} of the TG. Moreover, in order to check the parameter significance of the models, the necessary F-tests have been performed. If we assume that we need to check *i* parameters of the model for their significance, then the matrix of the unknowns in the system of observation equations can be written as

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_I \\ \mathbf{x}_{(I)} \end{bmatrix}. \tag{21}$$

where $\boldsymbol{x_I}$ are the parameter to be checked and $\boldsymbol{x_{(I)}}$ the rest of the model parameters. The check is performed for the null hypothesis $\boldsymbol{H_o}$***:*** $\boldsymbol{x_I=0}$ over the alternative one $\boldsymbol{H_o}$***:*** $\boldsymbol{x_I \neq 0}$ by determining the F-statistic (Dermanis and Rossikopoulos, 1991)

$$\tilde{F} = \frac{\hat{\mathbf{x}}_I^T \mathbf{Q}_{\hat{\mathbf{x}}_I}^{-1} \hat{\mathbf{x}}_I}{k\hat{\sigma}^2}, \tag{22}$$

where $\mathbf{Q}_{\hat{\mathbf{x}}_I}^{-1}$ is the sub-matrix of $\mathbf{Q}_{\hat{\mathbf{x}}} = N^{-1}$, which refers to the parameters $\boldsymbol{x_I}$ that need to be checked, and *k* is the number of parameters to be checked. The null hypothesis is accepted when

$$\tilde{F} \leq F_{k,f}^{a}, \tag{23}$$

where $F_{k,f}^{a}$ is the value of the *F* distribution, which we can derived from the standard statistical tables for given confidence level and degrees of freedom. For the present study we have used for all performed F-tests a standard confidence level of 95%. If Eq. (23) holds, then the parameters under investigation are deemed as not significant and are removed from the model, while in the opposite case they are retained. For the coefficients estimated from each model, a correlation analysis has been performed as well, in order to check if some of the parameters estimated are correlated among each other. The correlation between the estimated parameters was computed as (Bendat and Piersol, 2000):

$$\rho_{x_i x_j} = \frac{\mathbf{C}_{x_i x_j}}{\sqrt{\mathbf{C}_{x_i x_i} \mathbf{C}_{x_j x_j}}}, \tag{24}$$

where, $\mathbf{C}_{x_i x_j}$ denotes the cross-covariance between the parameters x_i and x_j, $\mathbf{C}_{x_i x_i}$ is the covariance parameter x_i and $\mathbf{C}_{x_j x_j}$ is the covariance of parameter x_j. Note that the covariances and cross-covariances are those estimated from Eq. (18), i.e., the covariance matrix of the adjusted unknowns. Another statistical test that has been performed in the frame of this study was the estimation of the simple and adjusted coefficient of determination (Rao and Kleefe, 1989), which both assess the goodness of fit. The simple coefficient of determination is given as

$$R^2 = 1 - \frac{\left[\sum_{i=1}^{n} (b_i - \hat{b}_i)^2 \right]}{\left[\sum_{i=1}^{n} (b_i - \overline{b})^2 \right]}, \tag{25}$$

where, $\overline{b}$ is the mean value of the observations, $\hat{b}_i$ are the adjusted observations $\hat{b}_i = b_i - \hat{v}_i$ and n is the number of observations. In the extreme case that the fit of the model to the data is perfect then $\sum_{i=1}^{n} (b_i - \hat{b}_i)^2 = 0$ and finally $R^2=1$. In the opposite case that the residuals of the errors are big enough to approach the magnitude of the observations themselves around the mean value, then the second term in Eq. (25) would be close to 1 and $R^2 \rightarrow 0$. Therefore the coefficient of determination ranges between $0 \leq R^2 \leq 1$ and the closer it is to one the better the fit of the model. Because R^2 is influenced greatly by the degrees of freedom and gives erroneously large values for parametric models with many parameters, the adjusted coefficient of determination can be used, which is free of the influence of the degrees of freedom (Fotopoulos, 2003)

$$R_a^2 = 1 - \frac{\left[\sum_{i=1}^{n} (b_i - \hat{b}_i)^2 \right] \Big/ (n-m)}{\left[\sum_{i=1}^{n} (b_i - \overline{b})^2 \right] \Big/ (n-1)}, \tag{26}$$

The final criterion which was used in order to assess the performance of the parametric models used, was the condition number that is determined as the ratio between the maximum and minimum eigenvalues of the matrix $\boldsymbol{A^T A}$, i.e.,:

$$con = \frac{\lambda_{\max}}{\lambda_{\min}}. \tag{27}$$

The largest the values of the condition number are the more unstable the parametric model and the results of the prediction are more susceptible to change with a change of the observations.

In the aforementioned, the problem for the homogenization of the Greek VRS has been formulated and the statistical tools that will be used in order to assess the results achieved have been presented. The data used in this combined adjustment scheme are sea surface topography measurements from available TG stations and a sea surface topography model (denoted as $\varsigma_c^{MSL\,i}$ and ς_c^i respectively in Eq. (4)). Figure 3 depicts the sea surface topography model used for the adjustment, which was determined from a combination of altimetric and marine gravity data, while its statistics are reported in Table 1. Its values range from about -17 cm in southeast Greece, where the Rhodes anticyclone is located, to 10 cm over the mid-Mediterranean current in the northeast with a standard deviation (std) of 5 cm.

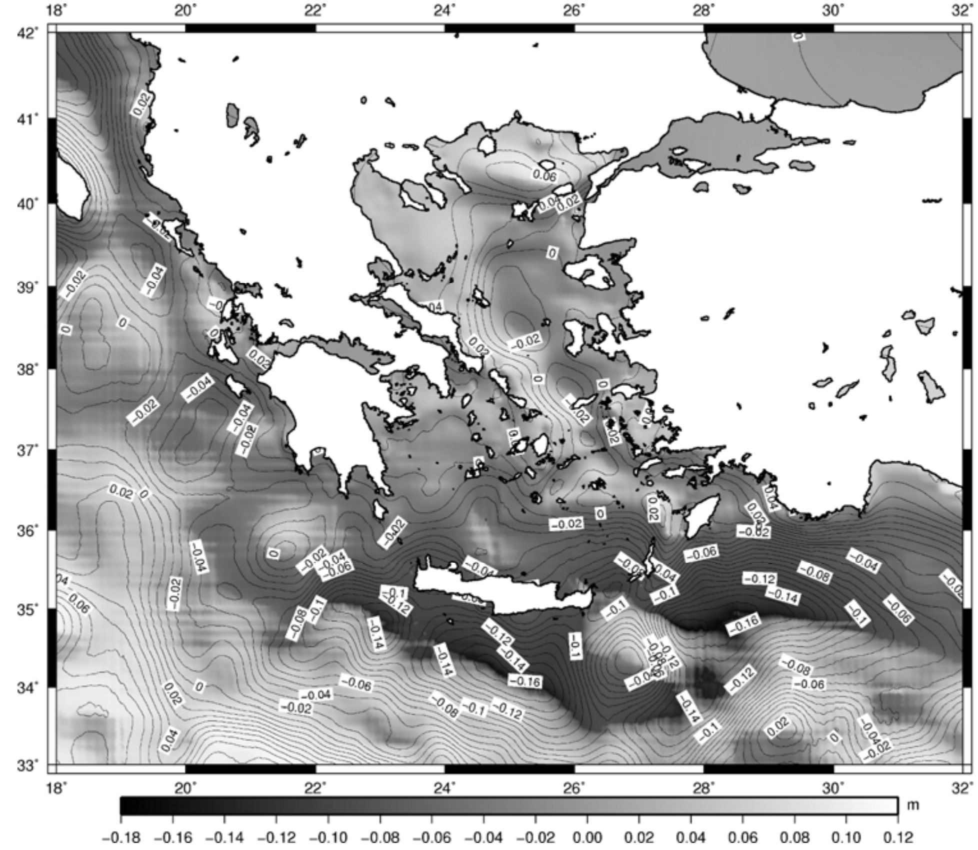

Figure 3: The sea surface topography model for the area under study.

Table 1: Statistics of the original SST model from the combination of altimetric and gravimetric geoid models. Unit: [m].

	max	min	mean	rms	std
ς_c	0.099	-0.176	-0.008	±0.139	±0.050

The TG stations for which data were available are presented in Figure 4, where it can be seen that they cover all mainland Greece and in all directions. It should be noted that the TG stations used in the present study are in continuous operation for more than 40 years (roughly from 1969 onwards), therefore they sample the full luni-solar tide of 18.6 years. Moreover, they are all located in harbours and they are built on concrete docks, while they are maintained by the Hellenic Navy Hydrographic Service. With these in mind it is assumed that both the TG MSL records as well as their levelling connections with the BMs do not contain any tidal effects or stability-related errors. The numbers in parentheses in each TG station indicate their $\varsigma_c^{MSL\ i}$ value that results from the available data according to

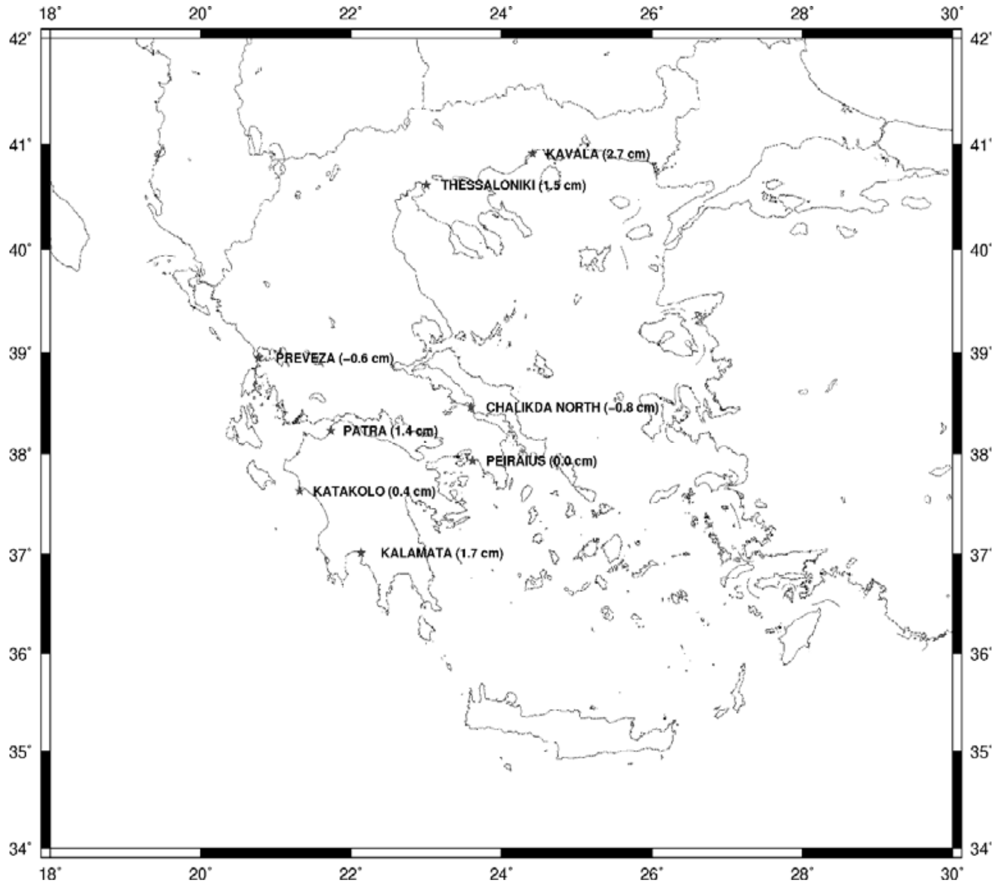

Figure 4: Distribution of the TG stations used for the height unification.

Table 2: Statistics of data at TGs for the datum homogenization. Unit: [m]

	MSL	**$\Delta H_{TG\text{-}BM}$**	**H_{BM}**	ς_{MSL}	ς_{c}
THESS	0.941	0.984	1.940	0.015	0.038
PIRAEUS	1.349	0.834	2.183	0.000	0.012
CHALKIDA	0.941	-0.122	0.811	-0.008	0.023
KALAMATA	1.440	-0.319	1.138	0.017	-0.001
KATAKOLO	0.859	1.639	2.502	0.004	-0.003
PATRA	0.720	1.964	2.698	0.014	0.011
PREVEZA	0.989	0.973	1.956	-0.006	-0.019
KAVALA	0.739	1.518	2.284	0.027	0.043

$H_i^{BM} - \Delta H_i^{TG-BM} - H_i^{MSL}$. The detailed statistics for each TG station are given in Table 2, where a) the MSL data, b) the levelling connection to the BM, and c) the orthometric height of the BM w.r.t. to Piraeus are reported. Note that the third column of Table 2 reports in all cases the Helmert orthometric heights of the tide gauge stations used. The last column in Table 2 reports the QSST values for the TGs from the available model, which were estimated with a bilinear interpolation. From the combination of these data and based on the scheme previously described, a new adjusted QSST model has been determined in order to homogenize the Greek VRS

3. Homogenization of the Greek vertical datum

As mentioned in the previous section, a variety of parametric models have been tested in order to select the one that provides the best fit to the data of the TG stations in Greece as well as the smallest prediction errors. From the various models tested the ones that will be reported herein are the 2^{nd} order polynomial one, the three-, four- and five-parameter similarity transformation and the QSST-dependant bias and scale ones. A zero order, corresponding to a mean removal, and a 1^{st} order polynomial model have also been tested, but the results are inferior to those of the other models and are not reported. Table 3 presents the coefficients of the corrector surface, the condition numbers and the simple and adjusted coefficients of determination that were computed for the various models. The 2^{nd} order polynomial model gives a good adjusted coefficient of determination at the 0.80 level with the condition number being about 6×10^4. Compared to the 3-parameter transformation model with an adjusted coefficient of determination at the 0.67 it performs better, showing that it fits better to the available data. The smaller condition number of the latter, being about 2×10^4, indicates that its solution is more stable than that of the polynomial model, but since both are of the same order of magnitude it can be concluded that the similarity transformation model is inferior to the polynomial one. This is even more apparent from the prediction errors reported in Table 4, where the errors of the 3-parameter model are generally larger than those of the polynomial model. Note that for some stations their differences in the prediction errors are as much as 1 cm (station KAVALA) and are very close to the signal of the QSST (2.7 cm for that station). It can be concluded that these two parametric models manage to provide a solution for the datum homogenization in Greece, but their performance is not sufficient to say the least. The same results hold for the QSST-dependant bias and scale model as given by Eq. (9), since the adjusted coefficient of determination is at the 0.67 level (last column in Table 3) and the prediction errors are quite large.

From both Tables 3 and 4 it becomes evident that the best results are acquired for the four- and five-parameter similarity transformation models. Both of these models have an adjusted coefficient of determination close to 1 (0.90), which indicates that their fit to the available data is almost perfect. Their condition numbers are larger than those of the lower order models being at the 5×10^8 and the 9×10^7 level respectively. These large condition numbers indicate that the results of the adjustment could be unstable and that the design matrices are not well-conditioned. The

Table 3: Coefficients of the corrector surfaces, condition numbers, coefficients of determination and adjusted coefficients of determination for the various parametric models.

2^{nd} order polynomial model
$x_o = -0.0083$ $x_1 = -0.0129$ $x_2 = -0.0061$ $x_3 = 0.0063$
$R^2 = 0.79$ $R^2_a = 0.78$ $con = 5.99\times10^4$
3-parameter similarity transformation model
$x_o = -0.2478$ $x_1 = 0.5536$ $x_2 = -0.5197$
$R^2 = 0.67$ $R^2_a = 0.66$ $con = 1.88\times10^4$
4-parameter similarity transformation model
$x_o = 38.8140$ $x_1 = -27.3822$ $x_2 = -12.2316$ $x_3 = -24.6888$
$R^2 = 0.91$ $R^2_a = 0.90$ $con = 4.59\times10^8$
5-parameter similarity transformation model
$x_o = 46.8390$ $x_1 = -36.5437$ $x_2 = -16.0076$ $x_3 = -17.6524$ $x_4 = -12.0030$
$R^2 = 0.91$ $R^2_a = 0.90$ $con = 9.26\times10^7$
QSST dependant bias and scale model
$x_o = 0.0044$ $x_1 = -0.7001$
$R^2 = 0.68$ $R^2_a = 0.67$ $con = 2.50\times10^3$

Table 4: Prediction errors for the various parametric models

	2^{nd} order polynomial	**3-param. model**	**4-param. model**	**5-param. model**	**QSST-dependant**
THESS	-2.8 mm	-7.6 mm	-3.6 mm	-1.1 mm	-0.8 mm
PIRAEUS	+9.2 mm	+2.9 mm	+7.4 mm	+2.1 mm	-5.0 mm
CHALKIDA	-13.6 mm	-18.5 mm	+7.7 mm	+4.2 mm	-11.6 mm
KALAMATA	+7.5 mm	+12.6 mm	+2.4 mm	+1.3 mm	12.9 mm
KATAKOLO	-9.0 mm	-2.9 mm	-7.2 mm	-3.1 mm	+1.1 mm
PATRA	-2.2 mm	-8.5 mm	-5.5 mm	+3.0 mm	+6.3 mm
PREVEZA	+6.7 mm	+2.5 mm	-3.7 mm	+1.9 mm	-4.7 mm
KAVALA	+4.3 mm	13.2 mm	-2.1 mm	+0.8 mm	+9.7 mm

problem of the high-condition numbers for the parametric models with the base functions selected (all depend on the geographic location of the available data and are combinations of the trigonometric numbers of the coordinates of the available point data) is known. Similar, and higher to the order of 10^9 - 10^{11}, condition numbers have been found in other areas too during the adjustment of GPS/Levelling geoid height data (Fotopoulos, 2003), signalling that with the more parameters added to the model the more unstable it becomes. Nevertheless, the prediction errors that these higher order models provide are small, so that we can conclude that their performance, even with the presence of a high condition number, is suitable for the Greek VRS homogenization. Note that especially the five-parameter similarity transformation model provides prediction errors which are at the few mm level for most stations (see Table 4, last column). The only stations with a high prediction error for the five-parameter similarity transformation model are those at the TGs of CHALKIDA and PATRA. But, from the TG distribution in Figure 4, it becomes evident that these two stations are the only ones which are between dry land areas, therefore in places that altimetry itself has great limitations and its observations contain larger errors compared to those in purely marine areas. Even for these two stations though, the prediction errors of the five-parameter model are the smallest ones, compared to the other choices tested.

Given that the five-parameter model has a smaller condition number as well, compared to the four-parameter model, and that it provides the smallest prediction error and an adjusted coefficient of determination close to 1, it was the one selected in order to determine the adjusted QSST model that was used for the homogenization of the Greek VRS. Even when comparing the five-parameter model to the QSST-dependant one (last two columns in Tables 3 and 4) it is evident that both the statistics of the linear system and the prediction errors that the former provides are superior. Note that the mean error for the five-parameter model for all TG stations is at the ±5.5 mm level, while for the QSST-dependant one it reaches the ±11.1 mm being twice as much. This is good proof that the results achieved with the 5-parameter model are the most rigorous ones and those that should be used for the unification of the Greek VRS. It should be noted that the correlation analysis performed for the coefficients estimated for all parametric models indicated that there is little or no correlation between them. Especially for the five-parameter similarity transformation model, which was the one finally selected for the unification of the Greek VRS, the correlation ranged between 10% and 22%. This was another proof, together with the performed F-tests, of the appropriateness of the parametric model selected. Figures 5 and 6 depict the corrector surfaces, i.e., the values $\mathbf{a}^T\hat{\mathbf{x}}$ for the entire Greek territory. It should be noted that for all models the necessary F-test described in the previous section has been performed for all parameters and in all cases they were deemed significant.

Based on the results acquired and the selection of the five-parameter similarity transformation model as the one to be used, the final adjusted model of the sea surface topography has been estimated using Eq. 20. Table 5 presents the statistics of the adjusted QSST model, while Figure 7 gives a representation of its values. This model, with a range between -10 and 10 cm and a std of ±2.4 cm for the entire Greek territory, provides the necessary *correction* values that should be applied to the MSL of TG stations, both for insular and mainland regions, so that their zero level will refer to that at the Piraeus TG station which forms the origin of the Greek VRS. Note, that due to the distribution of the available TG data, the corrections that this model provides might be unreliable in the edges of the country, i.e., in the

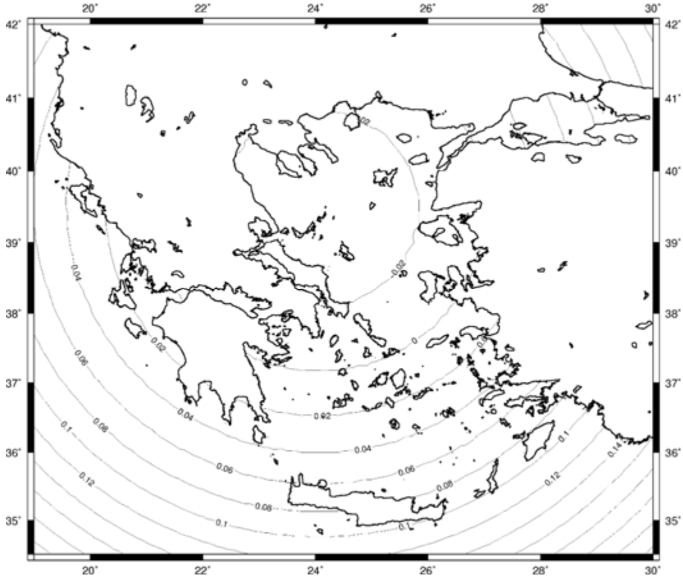

Figure 5: The corrector surface from the four-parameter similarity transformation model.

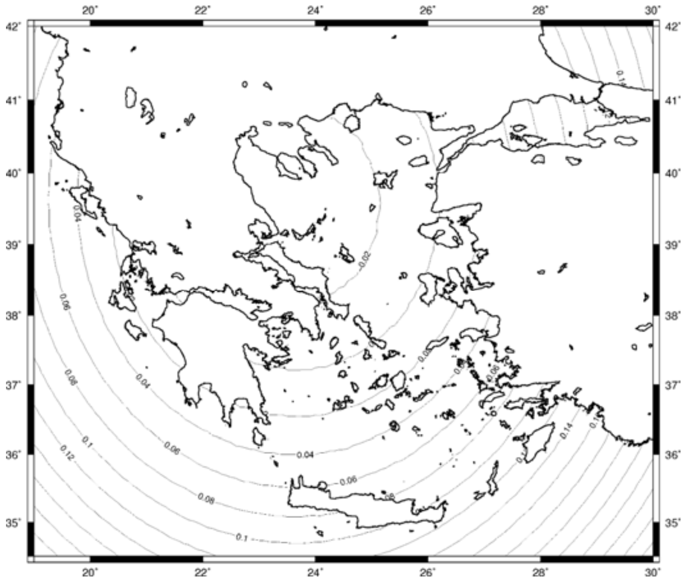

Figure 6: The corrector surface from the five-parameter similarity transformation model.

southeast part of the Aegean Sea. After applying this correction, either positive or negative, to a TG station MSL data, it will be referred to the W_0 of the Greek VRS as determined by the Piraeus TG, and additionally all BMs whose orthometric heights have been determined with respect to the local MSL of this TG station will refer from now on to the country's origin. In this way, the Greek vertical reference

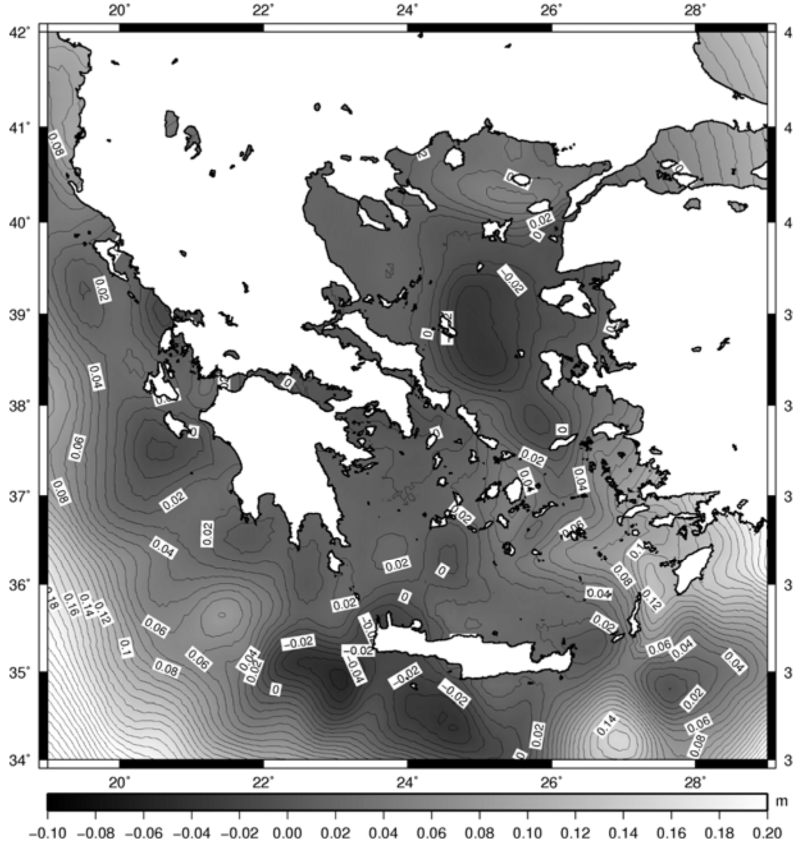

Figure 7: The final corrector surface for the unification of the Greek vertical datum. Values are corrections to be applied at TG records to be referred to Piraeus.

system can be unified and refer from now on to a common origin and zero level which will be that of the Piraeus station. Finally, when this unification is performed, then it will be possible to include the Greek levelling data to the United European Levelling Network and contribute to the combined adjustment of all European data towards the realization of the European Vertical Reference Frame and the connection of the country's levelling network to those of the other European countries. Unless such a unification of the Greek vertical system is performed, it will not be possible proceed to the unification with the rest of the European countries.

Table 5: Statistics of the adjusted SST model for the unification of the Greek vertical datum. Unit: [m].

	max	min	mean	rms	std
$\varsigma^{c\ adj}$	0.096	-0.094	0.011	±0.026	±0.024

4. Conclusions

A detailed description of the problems related to the homogenization of the Greek vertical datum have been presented, with the main ones being the lack of levelling connection between the insular and continental part of the country along with fact no common adjustment of the vertical network has been performed. In effect this means that the orthometric heights at an island refer to the MSL of a local TG station as that is determined by its own record of measurements, i.e., to a different zero level W_0^{local}, and are not connected to the rest of the country and more particularly to the zero level realized by the Piraeus TG station measurements. For the continental part of the counrty, since the vertical network has not been uniformly adjusted in a single step and the fact that some mainland areas do not refer to the origin of the country's vertical datum, but to some TG station that resides nearby, biases exist even between neighbouring stations. As a result of the aforementioned problems, Greece does not have a unified vertical network with all orthometric heights referring to the same zero level of the supposed origin, which is located in the TG station at the Piraeus harbour. This presents clear drawbacks for the usual surveying, geodetic and engineering in general projects, while the Greek VRS data cannot also be included in the realization of the EVRS since they are not consistent even among each other.

With these problems in mind, and with the fact that the deviation of a local TG zero level to that of the zero level of the TG at Piraeus harbour can be simply expressed as the quasi-stationary part of the sea surface topography, a combined adjustment scheme has been presented for the homogenization of the Greek VRS. The data used, refer to the TG MSL records, their levelling connection to a BM whose orthometric height is known w.r.t. to the zero level of the country and a mean dynamic topography model, which was available and covers the entire Greek territory. The latter was determined through an optimal combination of satellite altimetry and marine gravity data in the frame of a sea surface topography and ocean circulation study for the Aegean Sea. Various tests for the adjustment were performed by considering the performance, goodness of fit and prediction errors of a number of parametric models, ranging from low-order to high-order ones.

From the results acquired it became evident that the lower order models like the 2nd order polynomial and the three-parameter similarity transformation one provided relatively small condition numbers, but the adjusted coefficients of determination were considerable smaller than those of the higher-order models. The most important reasoning that deems them as inappropriate for the adjustment are the large prediction errors they provide, which almost at the level of the signal that needs to be predicted. The four- and five-parameter similarity transformation models provided similar results as far as the adjusted coefficients of determination are concerned, while the latter gave a smaller condition number, showing that it is more stable than the former. Moreover, the five-parameter model provided smaller prediction errors for all stations, so that finally it was the one selected to be used for estimation of the adjusted sea surface topography model. This adjusted QSST model provides *correction* values to a TG station MSL data, so that it will now refer to the W_0 of the Greek VRS as determined by the Piraeus TG. In this way, the Greek vertical reference system can now be unified and refer from now on to a common origin and zero level. It is acknowledged that the accuracy of the data used, and mainly that of the SST model, is within the limits of the signal that needs to be modelled, mainly due to the errors in altimetric observations close to the coastline. Nevertheless, this is a first attempt of the unification of the Greek vertical datum and probably the most rigorous and accurate one that can be performed with the data available today. Significant improvement can only be expected with the incorporation of GOCE data in the determination of the sea surface topography and the marine geoid along with the availability of GPS measurements at the TG stations.

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