Possible model of electromagnetic signals before earthquakes

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Abstract: Few days before many earthquakes a general change in the ratio of vertical to horizontal magnetic field components in the ULF band, often called "polarization", has been observed that can be considered as a magnetic precursor of the subsequent earthquake. To explain such a specific behavior, we propose a simple model based on a linear current approximation depending on some assumption on the signal to noise ratio. This model is confirmed through a comparison that has been carried out between the experimental magnetic field data at Matsushiro (Japan, 1998.06.30, M=4.7) and Simeiz (Crimea, Ukraine, 1998.10.16, M=4.3 and 1998.10.18, M=4.3) earthquakes. About 1 – 2 days before these events we had recorded the anomalous decrease of the ratio of vertical to horizontal magnetic field components in Pc4 – Pc3 band. The NE-SW direction of the corresponding current linear model well agrees with the main tectonic feature of both seismogenic events. Additionally we also estimate the signal to noise ratio limits for the detection of ULF magnetic field components in Corralitos and Stanford campus for Loma Prieta (M_S 7.1, 1989) earthquake.

1. Introduction

Many earthquakes (EQs) occurring at different parts of the Globe have high destructive force and are very dangerous for the populated and industrial areas. Since the EQs have undetermined time and location, the study of their potential precursors is very important. Among different types of EQ precursors the electromagnetic (EM) ones seem to be the most reliable (e.g., Asada et al., 2001; Hayakawa and Molchanov, 2002).

However these precursors are very different in their peculiarities and they vary not only at diverse places but also for different time even in the same place (Johnston, 1997; Park et al., 1993). Apparently this fact depends on the specifics of geological and tectonic settings and on the types of EM signal source in seismoactive zones (e.g., Hayakawa et al., 2000). For the most uncertainty of detailed object picture, a progress in EQ prediction practice using EM signatures has been very modest.

According to many experimental data, most often the precursors appear in the ULF

(0.01-10 Hz) band. For instance, Fraser-Smith et al. (1990) recorded anomalous magnetic field variations about two weeks before the 17 October 1989 Loma Prieta Ms=7.1 EQ in central California. Other anomalous ULF fluctuations possibly related to EQs were observed several hours prior to the 7 December 1988 Ms=6.9 EQ in Spitak, Armenia, (Molchanov et al., 1992, Kopytenko et al., 1993), and both about two weeks and a few days before the 8 August 1993 Ms=8.0 Guam EQ (Hayakawa et al., 1996). These anomalous ULF signals were always detected in an area close to the epicentre: far from the epicentre no anomalous signal can be observed due to the attenuation of the magnetic signal with distance.

This paper is an attempt to explain some experimental facts specific for seismogenic ULF emissions and to propose a possible configuration of the EM source signal. We shall concentrate on understanding and modelling the general decrease of the horizontal to vertical magnetic components ratio. The necessity of the study is called forth by the fact that this ratio, often called "polarization", is still reported to be observed as a precursor of the main shock in the vicinity of the epicentral area (Stanica and Stanica, 2012). We may assume that this feature is typical for magnetic pulsations of the lithosphere and it can be used as a distinctive characteristic of seismogenic EM signal, but in no way may be used as the EQ precursor. We will propose here a simple linear current model that explains this phenomenon.

The paper is organized as follows: after this introduction we propose the model, in the successive section we see two cases of study to which we apply our model and describe some evidences, finally we discuss the results and make some concluding remarks.

2. The model of Magnetic Field For Linear Current Approximation

All magnetic field variations connected with EQ activity are caused by some electrical current flowing during the stress deformation in the crustal structures, though the nature of current sources is very controversial (Draganov et al., 1991; Fenoglio et al., 1995; Johnston, 1997; Ogawa and Utada, 2000; Egbert, 2002; Hayakawa and Hattori, 2004). As is widely accepted now (e.g., Dudkin et al., 2003), these currents may be resulting from: 1) movement of a conductive medium (mostly fluids) in the Earth's magnetic field (inductive effect), 2) displacements of boundaries between highly and low conductive crustal areas (in fact it leads to the previous effect), 3) piezoelectric or piezomagnetic effects.

Let us consider the linear source of electrical current that may be extended (for example along the rift zone) or short (local fault or deformation). For extended source the following conditions take place: $L \geq 3\rho$ and diameter D $\leq \rho/3$, where L is the length of current I with spatial density J ($I = JC$), C is the average cross-section of the current tube, diameter D is the maximum linear dimensions of current tube cross-section, and ρ is the distance to an observation point. We expect that the direction of the equivalent current that can explain the anomalous magnetic field

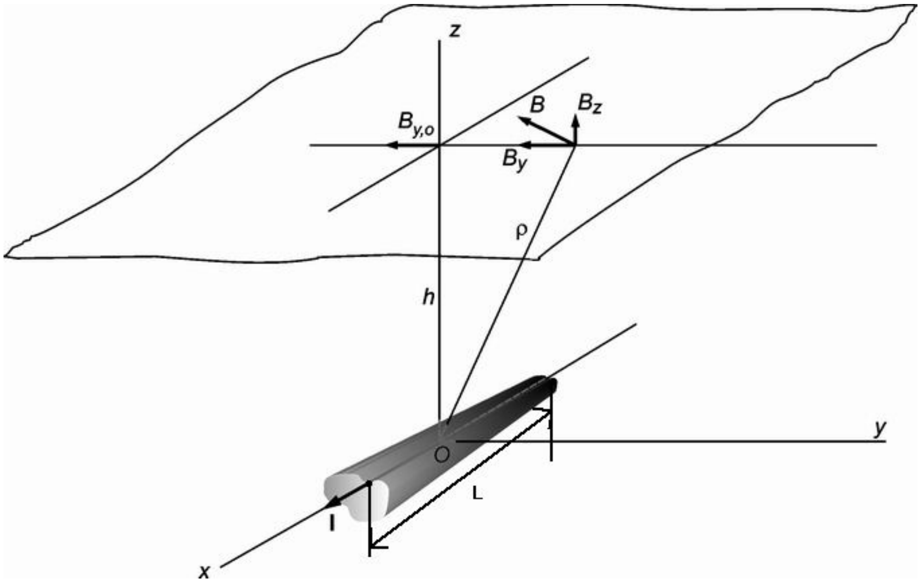


Figure 1 A linear current model for the estimation of anomalous magnetic field values.

fluctuation is geometrically similar to the direction of tectonic structure, which is in a process of the preparation to the main shock (Kulchitsky et al., 2004; Bortnik et al., 2010). This means that although the final dynamics of the EQ can follow a different direction we do not exclude that the current line might be aligned with the main tectonic direction that characterizes the seismic event: this will be actually confirmed by the results we will show in the next sections.

The magnetic field by such a linear source, in quasi-static approximation, can be described by the field of a long linear current:

$$\left.\begin{array}{l} B_2(\mathrm{nT}) = 200Ih/\rho^2 \\ B_3(\mathrm{nT}) = 200Iy/\rho^2 \end{array}\right\}, \tag{1}$$

where $\rho = (y^2+h^2)^{0.5}$, x, y, z are Cartesian coordinates; h is the depth of the current; I is oriented along x-axis (see Figure 1); $B_2 = B_y$, $B_3 = \mathrm{B_z}$ are given in nT while the other quantities are in SI units. Estimations of physical values orders are given in Appendix A.

It is clear from (1) that vertical component of the magnetic field near the vertical axis is close to zero. With increasing the distance y, the vertical component increases and when $y \approx h$, then $B_z \approx B_y$. Because all the field components quickly decrease with further increase in y, the detection of them is possible only for big enough signal to noise ratio (S/N). Let us accept:

$$B_{y,0} = (S/N)B_N\,, \tag{2}$$

where $B_{y,0}$ is the B_y value at $y=0$, B_N is the environmental magnetic noise level. We consider that to provide the reliable detection of the signal we must have:

$$B_{z(y)} / B_N \geq (S/N)_T , \tag{3}$$

where $(S/N)_T$ is the threshold (S/N) ratio (the minimum signal to noise ratio that allows us to detect the anomalous signal). Then from (1) and (2) we can determine the range of distances where the B_z or B_y component is detectable.

For $L \leq 3\rho$, the magnetic field of a source, in quasi-static approximation, can be described by the field of a dipole current (Wait, 1982):

$$B_i(\mathrm{nT}) = 100 r^{-3} \left(M_j x_k - M_k x_j \right), \tag{4}$$

where $i, j, k = 1, 2, 3$; $B_i = B_{x_i}$, $x_1 = x$, $x_2 = y$, $x_3 = z$, $r = (x^2+y^2+z^2)^{0.5}$, indices i, j, k form substitution: $\begin{pmatrix} 1\,2\,3 \\ 2\,3\,1 \\ 3\,1\,2 \end{pmatrix}$, $M_i = M_{x_i} = Ix_i$ is the projection of current moment $M = m_0 IL$ on the x_i axis, m_0 is the unit vector of M. Also here B_i are in nT while the other quantities are in SI units. As it is seen from (4), also the vertical magnetic component in the vicinity of the vertical axis of a local anomaly is close to zero. In the same way as in (2) and (3), we may introduce similar signal to noise ratios for a local source

$$\left. \begin{array}{l} B_{x(y),0} = (S/N) B_N , \\ B_i / B_N \geq (S/N)_T , \end{array} \right\} \tag{5}$$

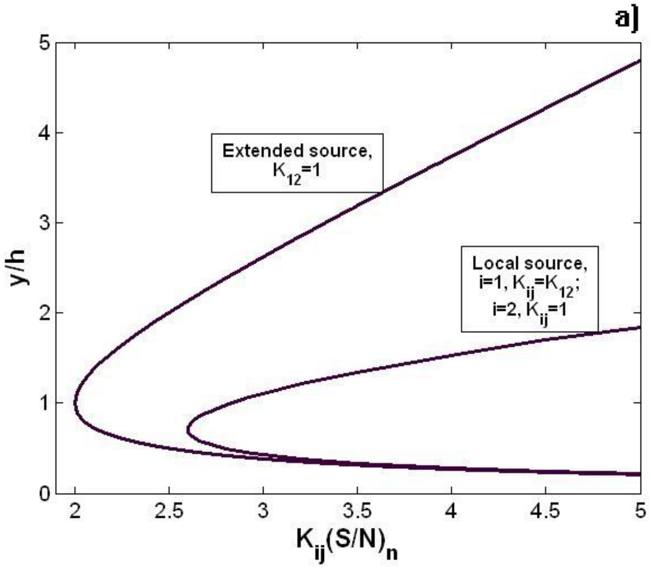
and we may determine respective ranges of distances for which the components of anomalous magnetic fields are detectable.

The results of calculation for these ranges of distances (in the normalized form of y/h) against the normalized signal to noise ratio [$(S/N)_n=(S/N)/(S/N)_T$] are shown in Figure 2a, b: a) for vertical and b) for horizontal components (see also Appendix B).

These ranges are within the corresponding curves both for extended and local sources in Figure 2a, and between the abscissa and corresponding curves in Figure 2b. For local sources $K_{ij} = M_i/M_j$, and for extended one $K_{ij} = 1$. Because the behavior of anomalous field with the movement of observation point along y and x axes is similar for the local source (see Eq. 4), we confine ourselves to considering only the case of y variation at $x = 0$. For the local source: in Figure 2a, $K_{ij} = K_{12}$ at i=1, and $K_{ij} = 1$ at i=2; Fig. 2b, $K_{ij} = K_{32} = 0.1$ or 1 at i=1, and $K_{ij} = 1$ at i=2.

The upper boundaries of normalized distance of signal detection for the extended and local ($i = 2$) sources are shown in Figure 3.

In these curves $(S/N) = (S/N)_T$ is the point of observation and the signal can be detected only in the domain $y/h - (S/N)_n$ below them. Obviously at $y/h < 1$ (from the left side of the point of curves inflection $y/h = 1$) this curve shows the detection limit of horizontal components and at $y/h > 1$ this limit is shown for the vertical



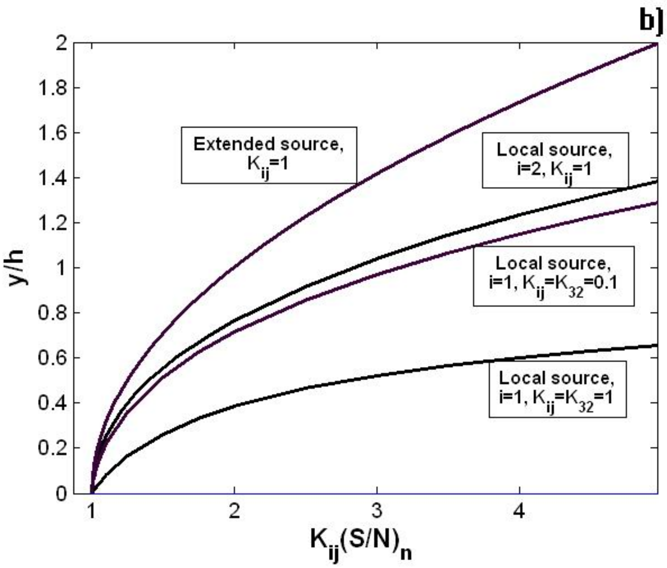


Figure 2 The normalized distance against normalized signal to noise ratio for both local and extended sources of anomalous magnetic field signals: a) for vertical component; b) for horizontal components.

one. The cases of successful (or unsuccessful) detection of ULF magnetic precursors can be described by the empirical correlation $y = f(M)$, where M is magnitude of EQ (Hayakawa and Hattori, 2004; Hattori et al., 2004). The correlation $y = f(M)$ is shown in Appendix Γ. We believe that most of these cases can be explained from our model of extended or local linear sources of precursors and (S/N) – ratio in the epicentral area.

3. Two case studies

To illustrate the application of our proposed model, we describe in this section the experiments for the study of ULF magnetic pre-EQ activity, which were carried out with two series of measurements made by LEMI-30 magnetometers (produced in Lviv Centre of Institute for Space Research) in the year of 1998. These magnetometers were placed in JMA's Matsushiro Seismological Observatory, Japan (geographic coordinates: 36,54^0 N, 138,21^0 E) (Hattori et al., 2004) and in Experimental Station of National Academy of Science of Ukraine at Simeiz, Crimea, Ukraine (44.4^0 N, 34.0^0 E). The magnetometers in Matsushiro Observatory were placed in a tunnel below the ground level at the depth of about 40 m. The location of the magnetometers in Simeiz was in seashore area. Both sites are in intensely populated areas characterized by high level of man-made interference, therefore only the late night-time (1.00 – 4.00 LT) was chosen for the analysis of the expected relatively weak seismogenic signals.

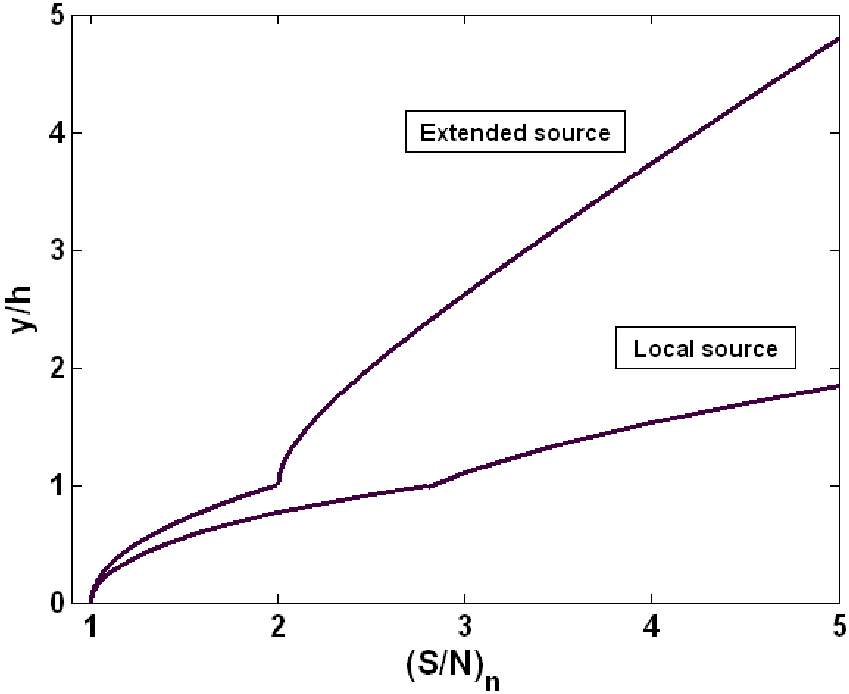


Figure 3 Normalized boundaries of signal detection against normalized signal to noise ratio.

All magnetic field components were analyzed in Pc4 – Pc3 frequency band (0.01 – 0.03 Hz). During the period of observation (June – October, 1998) three remarkable seismic events occurred relatively close to the described sites: one near Matsushiro (1998.06.30; 17.22.50 UT; 36.43^0 N; 137.72^0 E; h = 33 km; M=4.7) and two near Simeiz (Crimea, Ukraine, 1998.10.16, 15.24.10 UT, 44.04^0 N 33.61^0 E, h = 33 km, M=4.3; 1998.10.18, 05.22.10 UT, 44.03^0 N 33.62^0 E, h = 33 km, M=4.3). For all events, an anomalous behavior of the magnetic field components ratio B_z/B_i (i = x, y) was observed a few days before the EQs, happily in a period of very low magnetic activity.

On the 1-2 days preceding the EQs, the ratio of vertical to horizontal components of magnetic field decreased clearly in comparison with other days of observation what contradicts to the accepted suggestion that this ratio is increasing before the EQ (e.g., Hayakawa et al., 1996, 2002). As an example in **Figure 4** the ratio of $Z/(X^2+Y^2)^{0.5}$ for Matsushiro site is shown, where X, Y, Z are the components of magnetic field, time 0 corresponds 0 hours of local time on June 28, 1998. Corresponding magnetic activity was extremely low (daily $\sum K_p \leq 5.3$). These peculiarities were observed both in Matsushiro and in Simeiz. The possible model of such a strange picture of signals variation in these sites is considered below.

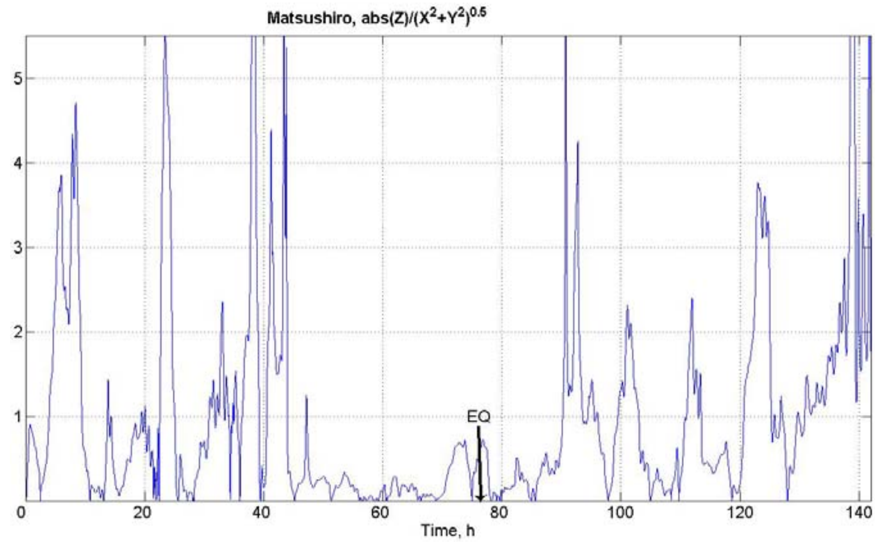


Figure 4 The ratio $Z/(X^2+Y^2)^{0.5}$ for pre-EQ ULF magnetic field near Matsushiro site is shown, where X, Y, Z are the components of magnetic field; abscissa is the local time of June 28, 1998.

4. Discussion of results

The obtained results can be useful for the estimation of possible type and location of anomalous magnetic field source at a given normalized signal to noise ratio. For practical reasons, we limited the $(S/N)_n$ – ratio in the vicinity of vertical axis over the source by values 1 – 5. It is worthwhile to note that at $(S/N)_n = 1$ the anomalous signal is detectable only in the close proximity to the source epicenter because of $(S/N) = (S/N)_T$

It is clear that at $(S/N)_n \leq 2$ for the extended source and at $(S/N)_n \leq 2.8$ for the local source, only the horizontal magnetic field components can be detected and only in the range of normalized distances $y/h \leq 1$. Although at $y/h > 1$ the detection of both anomalous variations of horizontal and vertical components is possible, however the upper limit of range for normalized distances at a given $(S/N)_n$ – ratio is higher for the vertical component than for horizontal ones. Obviously the extended source approximation can be applied for the condition of $y/h \leq ((L/3h)^2 - 1)^{0.5}$, thus for deep sources only at a narrow range of y/h.

For demonstration of obtained estimations, let us find the conditions on which the ULF magnetic precursor can be detected for Loma Prieta (M_S 7.1, 1989) EQ. The focal depths of two major subevents are 12 and 16 km (Choy and Boatwright, 1990). The ULF magnetic precursor exceeded the normal background level about 10 – 100 times in the Corralitos, California, 7 km from the epicenter and was not found in Stanford campus, 52 km from the epicenter (Fraser-Smith et al., 1990). If we suppose that the source was the local one and (S/N) – ratio was the same as in the epicenter, then for $(S/N)_T = 2$ we can find that the signal was undetectable for horizontal and vertical components at $(S/N) < 40$ in Stanford for $h = 12$ km and for $h = 16$ km at $(S/N) < 25$. This estimation can explain the stated effect if the error of (S/N) – ratio evaluation given above was rough enough.

It follows from Figure 2 and Figure 3 that for reasonable $(S/N)_n$ – ratio values (≤ 5) the upper limit of distances for the detection of anomalous magnetic field signals do not exceed $(3 – 4.8)h$ for extended sources and $(1 – 1.8)h$ for local sources. Besides this, the horizontal magnetic field components exceed the vertical ones at distances $y < h$.

Applying these results to both analyzed EQs at $(S/N)_T = 2$ and $y \approx 1.4h$ (Matsushiro), $y \approx 1.6h$ (Simeiz), we can conclude that: a) the considered anomalous magnetic field behavior was triggered by an extended or intermediate type of source ($L \geq \rho$) but not by a local one; b) probably, the current line of anomalous source was aligned close to the north–east south–west (NE-SW) direction, thus the y/h – ratio was less than 1 and the horizontal component of magnetic field prevailed over vertical one. Such a picture is rather atypical because the most of well-known ULF magnetic precursors were found at distances $y/h > 1$, where, according to our model, the vertical component has to prevail over horizontal one (see, for example: Hayakawa et al., 1996, 2002; Akinaga et al., 2001). The direction (NE-SW) found for both events is confirmed also from the corresponding focal mechanisms (e.g. Dysart et al., 1988, Iwata and Nakanishi, 2004 for Matsushiro seismogenic region and Giardini and Sellami, 1999 for Simeiz area) providing support to our model.

5. Concluding remarks

A simple current linear model has been proposed in this paper in order to explain the anomalous magnetic field components behavior often observed from a few days to some hours before a seismic event in the vicinity (within a few tens of km) of the epicentral area. It is rather clear from this model that such a feature as "polarization ratio" can not be used as EQ precursor. It should be noticed that our calculations are rather oversimplified because of magnetic field quasi-static approximation. However, they are clear enough and useful for the upper estimations for normalized distances. Taking into account appropriate considerations about the signal to noise ratio, the same model also explains why sometime there is no any anomalous magnetic field behavior. Applying this model to the magnetic field record observed before and during the series of EQs of magnitude greater than 4 occurring in 1998 in Ukraine and Japan, we find a characteristic lineation of the equivalent current that agrees well with the main tectonic direction which produced the seismic events. Although this fact is not always expected, since the preparation process that produces the anomalous magnetic field variation could have a different lineation from that of the main shock, the agreement between main directions of electromagnetic and seismic geometries is encouraging.

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APPENDIX A

Let us estimate the value of current density in the current tube with cross-section S=1km*1km for big enough the magnitude of precursor's magnetic field vector B=1 nT on the range ρ=100 km. From Biot-Savart law for extended linear current such a magnitude equals

$$B = \mu_0 I / (2\pi\rho), \tag{A1}$$

where μ_0 =4π*10^{-7} H/m is magnetic permeability of free space (or nonmagnetic medium).

In nanotesla units from (A1) it follows

$$B(\mathrm{nT}) = 10^9 4\pi 10^{-7} I / (2\pi\rho) = 200 I / \rho = 200 JS / \rho, \tag{A2}$$

where all values in right side of (A2) are in SI units.

From (A2) for given values we get

$$J = 5\times 10^{-8}\,\mathrm{A/cm^2}.$$

Such a value of current density in earthquake preparation area is modest enough and can be basic for geophysical models of magnetic precursor source. Similarly for short (or dipolar) sources we get an upper limit of current density and the estimation for sources of intermediate length will be between these limits.

APPENDIX B

For explaining the used approach, let us find the connection between $(S/N)_n$ –ratio and normalized distance y/h for a local source at $x = 0$. It follows from Eq. 4 that at the vertical axis where $x = 0$, $y = 0$ we have

$$\left.\begin{aligned} B_{x,0}(\mathrm{nT}) &= 100 M_y / h^2, \\ B_{y,0}(\mathrm{nT}) &= 100 M_x / h^2 \end{aligned}\right\}. \tag{B1}$$

From (4) and (B1) we can find

$$\left.\begin{aligned} B_z / B_{x,0} &= h^2 (yK_{12} - x) / r^3, \\ B_z / B_{y,0} &= h^2 (xK_{21} - y) / r^3. \end{aligned}\right\} \tag{B2}$$

Substituting (5) in (B2) at $x = 0$ we get the equations connecting the required values

$$\left.\begin{aligned} u_y^6 + 3u_y^4 + \left(3 - (K_{12}a)^2\right) u_y^2 + 1 = 0, \\ u_y^6 + 3u_y^4 + \left(3 - a^2\right) u_y^2 + 1 = 0, \end{aligned}\right\}, \tag{B3}$$

where $u_y = h/y$, $a = (S/N)_n$, and $K_{12} = M_1/M_2 = M_x/M_y$. The general view of Eq. B3 for the case $x_k = 0$ is shown below

$$u_l^6 + 3u_l^4 + \left(3 - b_{kl}^2\right)u_l^2 + 1 = 0\,, \tag{A6}$$

where $k, l = 1, 2$; $b_{kl} = aK_{kl}$. Other equations for the estimation of required values are calculated in the same way.

APPENDIX Γ

For convenience of article reading in Fig. A we quote the empirical correlation $y = f(M)$, where M is magnitude of EQ from (Hattori et al., 2004). Here the summary on the occurrence of the EQ-related ULF activity in the form of EQ magnitude versus epicentral distance from an ULF magnetic station is shown. White and black circles show an EQ with and without ULF anomalies, respectively. The dashed line indicates the empirical threshold for appearance of the anomalous ULF signals preceding large EQs. This figure demonstrates that ULF emissions could be observed about 60 km from the source region for an EQ with M≥6, and the detectable distance of ULF magnetic anomalies would be extended to about 100 km in the case of an EQ with M≥7.

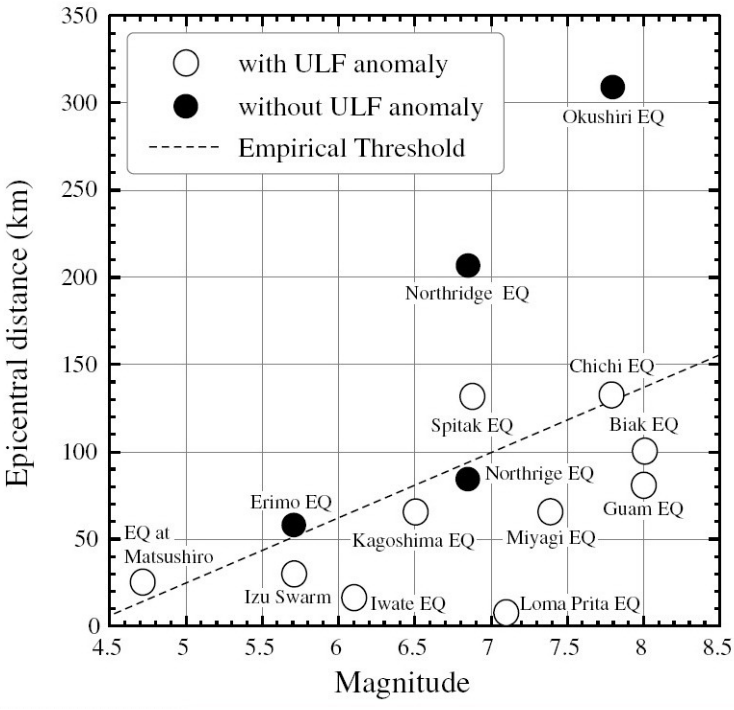

Figure Γ1. The ULF signature of EQs in the form of EQ magnitude versus epicentral distance (from Hattori et al., 2004).