Evaluation of the Hellenic Geodetic Reference System 1987

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Abstract: The paper presents a detailed accuracy evaluation of the Hellenic Geodetic Reference System 1987 (HGRS87), a non-geocentric traditional local datum which was established almost thirty years ago and it is still used as the official spatial coordinate system for surveying applications in Greece. This is the first time that a large-scale assessment of the HGRS87 frame is performed in a unified manner, which aims to reveal its weakness by quantifying its consistency with a higher quality ITRF/GNSS-based frame over the entire country. A national geodetic velocity model is employed in our analysis in an attempt to eliminate the accumulated geodynamical distortions of the (static) HGRS87 coordinates due to the non-uniform velocity field of the Hellenic area, and thus to provide a more judicious comparison between the underlying frames at a common epoch. Our results show that, even after the removal of the geodynamical distortions, the average accuracy of HGRS87 at national scale remains worse than 45 cm, yet significant variations exist in its regional accuracy over different parts of Greece.

1. Introduction

Greece is one of few countries that has not yet officially adopted a geocentric GNSS-based reference system to serve its national geodetic, surveying and mapping needs under a modernized geospatial framework. Despite some previous efforts towards this direction (Delikaraoglou 2008, Katsambalos et al. 2010) the geodetic reference system of Greece has not been updated during the last thirty years and it still relies on a non-geocentric traditional local datum named *Hellenic Geodetic Reference System 1987* (HGRS87). This is a horizontal coordinate system based on the GRS80 ellipsoid that was endorsed by professor G. Veis in the mid-eighties and realized over a terrestrial network of more than 25,000 benchmarks (BMs) via the joint work of the Hellenic Military Geographic Service and the National Technical University of Athens (Takos 1989, Veis 1995). Its name designates the year that it was formally adopted by the national mapping and cadastre agency (HMCA 1987) although its public use for surveying applications did not fully commence before the early nineties. At present, most of the professional work by licensed surveyors in Greece, including land demarcation, cadastral mapping and boundary surveying, is mandatory to be referenced and delivered to the HGRS87 system. This is accomplished either by terrestrial observations to existing

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BMs or by combining GNSS observations with a nation-wide coordinate transformation model which is supported by all RTK reference networks in the country (Kotsakis and Katsambalos 2008, Katsambalos et al. 2010). Hence, in spite of its outdated character, the HGRS87 system not only remains an operational tool for handling the spatial data infrastructure in Greece but it will continue to play a key role as the official coordinate system for the on-going formation, and the prospective full operation, of the Hellenic cadastral system.

Similarly to most old national datums, HGRS87 suffers from large systematic errors and internal distortions in its established coordinate frame at spatial distances from few tens of kilometers up to few hundreds of kilometers. Considering its fragmented realization from multi-year terrestrial observations in separate blocks without following a unified procedure (Takos 1989), the HGRS87 accuracy is expected to vary between different regions in a rather complicated manner. In addition, it is well known that the Hellenic area undergoes considerable geodynamical deformation due to its non-uniform velocity field, thus causing relative crust movements up to a few centimeters per year between different parts of the country (e.g. Hollenstein et al. 2008, Floyd et al. 2010, Muller et al. 2013, Chatzinikos 2013). This inflicts additional distortions to the (static) HGRS87 frame whose accuracy level at national scale remains today largely unknown. Actually, the interference between the internal systematic errors with the spatially inhomogeneous velocity field has undermined the original HGRS87 realization, and it will continue to degrade its accuracy in both national and regional level.

The scope of this paper is to present a detailed accuracy evaluation of HGRS87 based on a large network of more than 2400 BMs with average spatial density of 1 point per 100 km^2 and well known positions in the latest ITRF2008 reference frame (Altamimi et al. 2011). The bulk of our test network, that is almost 98% of its stations, consists of third and fourth-order geodetic benchmarks which hold the major part of HGRS87's systematic distortions and they are commonly used as control points in surveying applications. Their spatial distribution covers the full extent of the national network, thus providing us with a reliable data ensemble for the purpose of our study. Note that this is the first time that such a large-scale assessment of the Hellenic geodetic reference frame is performed, aiming to reveal its weakness by quantifying its consistency with a higher quality ITRF-based frame over the entire country. A geodetically derived velocity model for Greece (Chatzinikos 2013) is employed in our analysis in an attempt to eliminate the accumulated errors in the static HGRS87 coordinates due to the non-uniform motion of the network stations – and thus to provide a more judicious comparison between the underlying frames at a common epoch. In this way, we are able not only to infer the original accuracy of HGRS87, but also to estimate its secular geodynamical distortion over different areas since its thirty-year old release in Greece.

2. The HGRS87 reference system and its realization

The establishment of HGRS87 is the result of a vast effort initiated in the early sixties by the Hellenic Military Geographic Service for the modernization of the geodetic infrastructure in support of surveying and mapping applications. This project took almost three decades to be fully completed (1962-1989) and it involved the re-observation and rigorous adjustment of the first and second-order geodetic networks, as well as their densification by new third and fourth-order networks which were observed and adjusted in separate blocks around the country.

According to Takos (1989), the primary realization of HGRS87 was obtained from the unified adjustment of the first-order network (137 stations) on the GRS80 ellipsoid using as fundamental reference point the central pillar at the Dionysos satellite observatory with fixed geodetic coordinates φ = 38° 4' 33".8, λ = 23° 55' 51".0 and geoid height N = 7.00 m. These values correspond to a geocentric offset of ΔX = 199.87 m, ΔY = –74.79 m, ΔZ = -246.62 m relative to the BTS87 (predecessor of ITRFs) and they were derived as a result of an optimal fit between the GRS80 ellipsoid and a satellite-based geoid model over the Hellenic area (Veis 1995). The data used for the first-order network adjustment included only terrestrial observations that were reduced on the GRS80 ellipsoid, and they had been collected over a time span of more than twenty years: 720 horizontal directions (1964-1979), 20 astronomical azimuths (1973-1986) and 17 baseline distances measured by geodimeters (1974-1979). A rotation and scale change in the order of 1 ppm was applied to the first-order solution to enforce its orientation and scale consistency with the BTS87 system. This additional correction was derived on the basis of 30 "zero-order" stations where Doppler measurements had been performed in the early eighties, resulting in precise estimation of their geocentric Cartesian coordinates (Takos 1989). The final geodetic coordinates at the 137 first-order stations from the aforesaid procedure provided the backbone for the subsequent densification of HGRS87 via second-order (475 stations), third-order (3903 stations) and fourth-order (21181 stations) networks, which are currently used for geodetic horizontal control in most surveying applications in Greece.

The realization in the lower order networks relied either on over-constrained adjustments of triangulation measurements over different regional blocks or on Helmert-type transformations of previously adjusted blocks in the so-called New Hellenic Datum (NHD) – which is based on the Bessel ellipsoid and a different fundamental point than the HGRS87 datum – that was co-realized at the same time by the Hellenic Military Geographic Service. The first approach was followed in the second-order network by dividing it into 80 polygons that were separately adjusted on the GRS80 ellipsoid by fixing a large number of HGRS87 first-order stations (and/or second-order stations which were already computed in neighboring blocks). The used data in this case included triangulation measurements from the period

1982-1989, which were augmented by 66 distance measurements (1975-1989) most of which covered the weakly triangulated parts in the northeastern region and the southeastern Aegean islands. The second approach was mostly employed in the third and fourth-order networks which were adjusted in the NHD also in a blockwise manner, and then transformed to HGRS87 based on known common stations (of various orders) in both datums. Although it is not clear which is the exact period of the triangulation measurements that were used in the latter densifications, the documentation by Takos (1989) implies that they had been performed earlier than the second-order measurements, most probably within the period 1962-1981.

Regarding the accuracy of HGRS87 it is generally claimed to be in the order of 4-5 ppm for first and second-order baselines and 10-15 ppm for third and fourth-order baselines (Takos 1989). These are internal quality indices obtained from the respective adjustments and the empirical analysis of the observation residuals, and they do not necessarily reflect the true HGRS87 accuracy throughout the country at different spatial scales. Some external evaluation work by the Hellenic Military Geographic Service in 1989 – based on the comparison of newly measured distances with computed distances from the official HGRS87 coordinates – showed a consistency level ranging from a few cm up to 20 cm (the worst performance occurred in the southeastern Aegean region); see Takos (1989). These evaluation results are not really informative since no mention is given regarding the length and the measuring accuracy of the tested baselines, the order of stations involved in the external evaluation and whether their HGRS87 coordinates had been obtained from the same block adjustment or not.

A most critical issue for our study is that HGRS87 was created by a combination of multi-year terrestrial observations without accounting for the irregular motion of the network stations. Given the large time span of data collection (1962-1989), this has resulted in considerable systematic errors due to inevitable inconsistencies among the various epochs of the lower-order measurements in relation to the reference epochs of the higher-order station coordinates that were kept fixed during the progression of the densification process. The identification of a single epoch to characterize the entire HGRS87 frame is a difficult and challenging task, thus making its comparison with a modern time-dependent frame (such as the ITRF) a non trivial problem. In the context of our study this complication is tackled via an empirical backdating procedure which will be analytically described in next sections.

Lastly, let us note that HGRS87 is a horizontal frame that is complemented by orthometric height information at all BMs, as obtained by leveling ties to the national vertical network during the time of its realization (Takos 1989). This additional height information is not used in the evaluation methodology which is followed in this paper.

3. The Hellenic geodetic velocity model

A geodetically derived velocity model for Greece was developed by Chatzinikos (2013) using daily GNSS data from 105 permanent stations of several control networks, including the Leica SmartNet network, the National Observatory of Athens (NOA) network, the HERMES network of the Aristotle University of Thessaloniki, the Hellenic Positioning System (HEPOS) network, and also 6 reference stations of the EUREF network. The used data cover a six-year period (2007-2013) with the majority of stations having full data availability of more than 36 months within this time range.

The aforementioned model describes the horizontal velocity field in the Hellenic area and it is derived from the time series analysis of estimated daily positions w.r.t. ITRF2008 frame (Altamimi et al. 2011). The velocity estimation for each station, along with the identification of hidden discontinuities in the coordinate time series, was performed with the H&D-MOGS (Hourly and Daily Monitoring of GNSS Stations) software that was developed in the Department of Geodesy and Surveying at the Aristotle University of Thessaloniki (Chatzinikos 2013, Chatzinikos et al. 2015). For stations with continuous data availability of more than two years, the horizontal velocities were estimated jointly with periodic (annual and semi-annual) signals that were also present in the coordinate time series. The magnitude of the estimated velocities in ITRF2008 range from 7 mm/yr to 26 mm/yr, with the smallest values occurring in the islands of Limnos and Lesvos whereas the largest appear at stations located in central Macedonia. The amplitudes of the periodic coordinate variations did not exceed 1.5 mm and 1 mm for the annual and semi-annual signals, respectively.

The estimated velocities reveal an inhomogeneous pattern of horizontal motion (see Figure 1) with relative crust movements between different parts of the network reaching up to 35 mm/yr. The most stable regions seem to be in the area of Macedonia while the central part of Greece and Peloponissos show significant velocity variations, both in terms of magnitude and orientation, over distances as short as few tens of kilometers. Obviously such geodynamical behavior has produced additional distortions in the (static) HGRS87 frame and it is responsible for degrading its accuracy level by almost 10 cm, at national scale, over the last twenty-five years (see next section).

The final velocity model, hereafter abbreviated as HGVM2013, was produced in gridded form with spatial resolution 15'×15' using a remove-restore interpolation scheme. Firstly, the entire network of 105 stations was divided into seven blocks in each of which a separate Euler-pole rotation model was fitted to the estimated velocities (see Figure 2a). The purpose of this preliminary step is to obtain residual station velocities that are as small as possible, in order to facilitate the minimization of their interpolation (gridding) error over the Hellenic area during the next

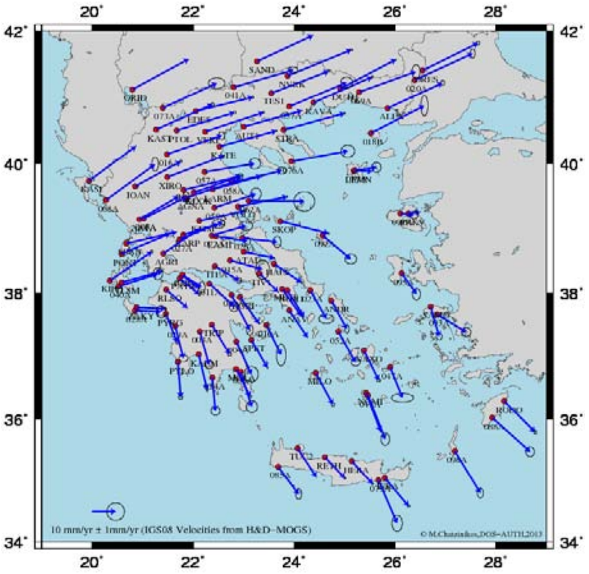

Figure 1. *The estimated horizontal velocities w.r.t. ITRF2008 at 105 permanent GNSS stations in Greece. Error ellipses correspond to 99% confidence level.*

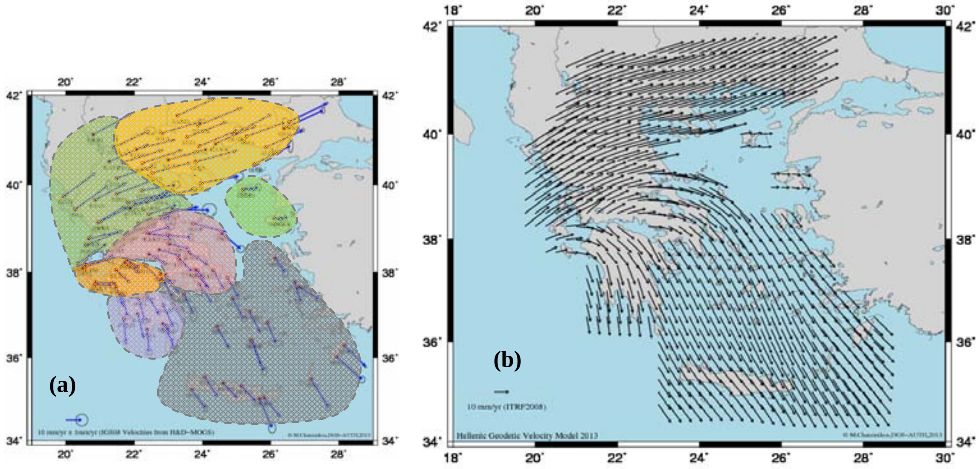

Figure 2. *(a) The seven blocks that were used in the Euler-pole fittings during the regional de-trending of the estimated station velocities, (b) the HGVM2013 grid model of the horizontal velocity field in Greece.*

step. The selection of the seven blocks did not rely on any geophysical criteria and it is based only on the actual behavior of the estimated velocities in the given network. The velocity residuals after the Euler-pole fittings were gridded, independently for each block, using least squares collocation with empirically determined covariance functions. At the last step, the effect of the Euler-pole rotation was restored back to the gridded residuals in order to obtain the total horizontal velocity model (see Figure 2b). For more details see Chatzinikos (2013) and Chatzinikos et al. (2015).

4. External evaluation of HGRS87

4.1 Data sets and methodology

The quality assessment of HGRS87 is based on multiple comparisons between two coordinate sets at 2431 BMs of the national geodetic network, see Figure 3. The first set consists of the official HGRS87 geodetic coordinates while the second contains the geocentric Cartesian coordinates w.r.t. ITRF2000 (Altamimi et al. 2002) at the epoch 2007.236. The latter were obtained from a dedicated GPS campaign that was organized by KTIMATOLOGIO S.A. during the setting up of the HEPOS system in Greece (Gianniou 2008). The one-sigma accuracy of these coordinates is 1-2 cm (horizontally) and 3-5 cm (vertically), and they were primarily used for developing the official transformation model between HGRS87 and the HEPOS operating reference system in support of GNSS surveying applications (Kotsakis and Katsambalos 2008, Katsambalos et al. 2010).

The direct comparison of the aforementioned coordinate sets is not possible since they refer to different reference systems. Furthermore, the ITRF coordinates reflect

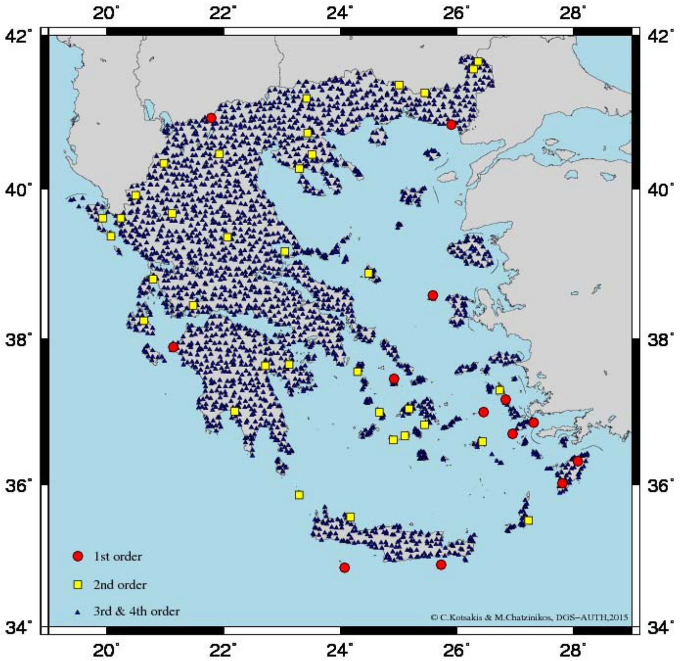

Figure 3. *The Hellenic test network (2431 BMs).*

the network geometry in the year 2007, which is distorted from the one implied by the first coordinate set due to the non-uniform station motions since the official release of HGRS87. To filter out the (secular part of) geodynamically induced discrepancies between the two coordinate sets, we used the HGVM2013 model to backdate the ITRF coordinates at a conventional epoch that best approximates the "reference epoch" of the HGRS87 realization in the test network. Subsequently, the backdated ITRF coordinates were fitted to the HGRS87 coordinates using a rigid (shift/rotation) transformation model to remove their datum differences, and the resulting residuals were used for an insightful analysis of the HGRS87 accuracy throughout Greece. The whole evaluation strategy is summarized in terms of the following steps:

1. Transform the geocentric Cartesian coordinates at the 2431 BMs from ITRF2000 to ITRF2008 (epoch 2007.236).
2. Choose a suitable "reference epoch" for the static HGRS87 frame using a least squares criterion for the residuals obtained by recursive transformations between the HGRS87 coordinates and yearly-backdated ITRF2008 coordinates at the 2431 BMs.
3. Backdate the ITRF2008 coordinates from step 1 to the reference epoch selected in step 2 by using the interpolated velocities from the HGVM2013 model.
4. Analyze the residuals from the frame transformation between the HGRS87 coordinates and the backdated ITRF2008 coordinates.

In the next sections we describe in more detail the above steps and their related results.

4.2 Helmert transformation of the ITRF coordinates and velocity interpolation at the test stations

The transformation from ITRF2000 to ITRF2008 is a necessary step in order to utilize the HGVM2013 velocity model for the coordinate backdating procedure in our test network. This step is implemented through the usual Helmert transformation model using the recommended parameter values by IERS which were reduced at the given epoch (2007.236) of the ITRF2000 coordinates.

Next, the ITRF2008 horizontal velocity at each BM was interpolated from the HGVM2013 model. The latter contains the topocentric velocity components, V_n and V_e, in the local NEU (north/east/up) system on the earth's surface. The interpolated velocities were converted to their geodetic counterparts w.r.t. the GRS80 ellipsoid using the following equations (Torge and Müller 2012):

$$V_\varphi = \frac{V_n}{M} \tag{1}$$

$$V_\lambda = \frac{V_e}{N\cos\varphi} \tag{2}$$

where φ is the geodetic latitude of each BM determined from its ITRF2008 coordinates while M and N denote the meridian and prime vertical radii of curvature of the GRS80 ellipsoid at the corresponding point.

The ITRF2008 coordinates of all stations were converted to geodetic coordinates w.r.t. the GRS80 ellipsoid and, in conjunction with the interpolated velocities from Eqs. (1) and (2), formed the basis for backdating the horizontal positions in our test network according to the formula

$$\begin{Bmatrix} \varphi(t) \\ \lambda(t) \end{Bmatrix} = \begin{Bmatrix} \varphi(2007.236) \\ \lambda(2007.236) \end{Bmatrix} + (t - 2007.236) \begin{Bmatrix} V_\varphi \\ V_\lambda \end{Bmatrix} \tag{3}$$

The choice problem of the (unknown) reference epoch t that corresponds to the HGRS87 realization is discussed in the next section.

4.3 Selection of the reference epoch for the HGRS87 coordinates

HGRS87 is a static frame that is not formally associated with a particular epoch for its horizontal coordinates. In the presence of continuous geodynamical deformation in the Hellenic area and considering the long-lasting process that was followed for the HGRS87 realization, it is rather difficult to identify a single epoch that could characterize in a rigorous sense the entire frame. Nonetheless, our operational preference for this study is to assign an empirical "reference epoch" based on a best-fitting criterion between the HGRS87 and ITRF2008 coordinates in the test network. The adopted strategy employs successive backdatings, in yearly intervals, of the ITRF2008 coordinates based on Eq. (3). The backdated coordinates are then combined with the official HGRS87 coordinates via a least squares adjustment of the linearized similarity transformation model according to the general form (see appendix):

$$\begin{Bmatrix} \varphi_{HGRS87} - \varphi(t) \\ \lambda_{HGRS87} - \lambda(t) \end{Bmatrix} = \mathrm{f}(t_x, t_y, t_z, \varepsilon_x, \varepsilon_y, \varepsilon_z, \delta s) + \begin{Bmatrix} e_\varphi(t) \\ e_\lambda(t) \end{Bmatrix} \tag{4}$$

The "reference epoch" of the HGRS87 coordinates is selected as the one for which the root mean square (rms) of the adjusted residuals, $e_\varphi(t)$ and $e_\lambda(t)$, is minimized over all network stations. The rationale for this selection lies on the hypothesis that the velocity model describes adequately the secular part of the crust movement over a longer period which extends back to the establishment of HGRS87. Under this assumption, the least squares adjustment of Eq. (4) is expected to give a continuously improved fit between the two coordinate sets, up to an epoch where the geodynamical distortions have been eliminated – in an average sense – from the backdated ITRF2008 coordinates. This epoch can be perceived as a "reference epoch" for the static HGRS87 coordinates in the sense of reaching an optimal agreement with the backdated ITRF2008 coordinates for the geometrical shape of the test network.

Three different scenarios were tested with the aforementioned strategy depending on the selected transformation parameters in Eq. (4). Specifically, we used a shift-only model (3 translations), a rigid transformation model (3 translations + 3 rotations) and a full similarity model (3 translations + 3 rotations + 1 scale factor). The results from each scenario in terms of the rms of the (vectorized) residuals in different epochs are shown in Figure 4.

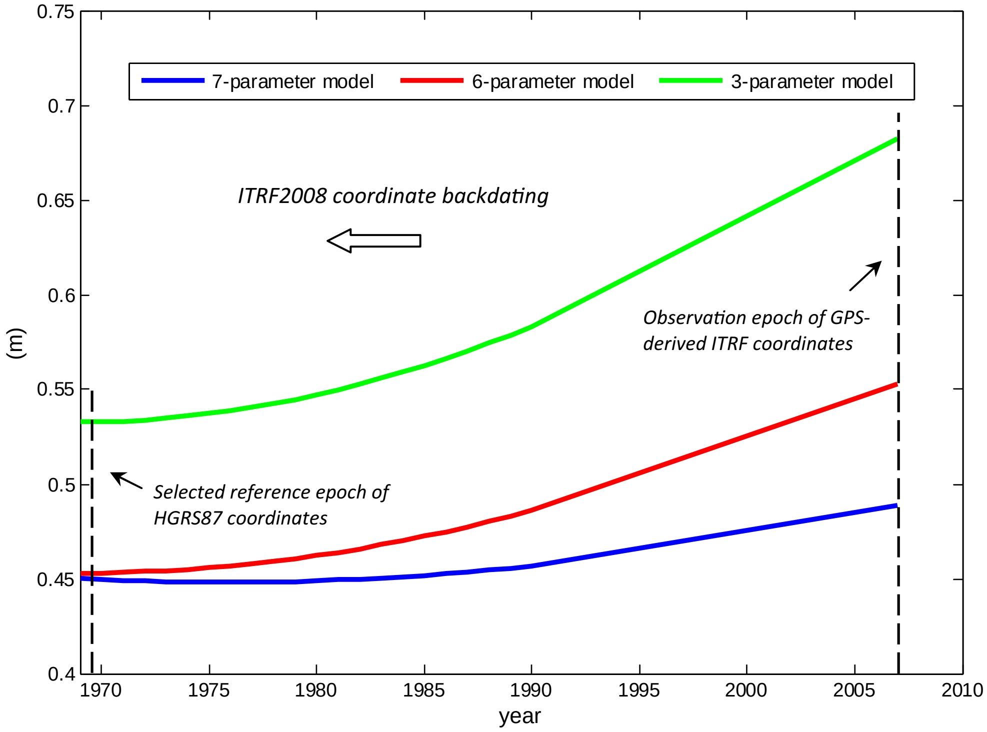

Figure 4. *Rms of the transformation residuals between the static HGRS87 and the backdated ITRF2008 coordinates by different models at various epochs. The rms refers to the magnitude of the vectorized residuals in the test network of 2431 BMs.*

Evidently, the coordinate backdating improves the fitting between HGRS87 and ITRF2008 in all transformation scenarios. The reduction rate of the residuals is larger over the last two decades (1987-2007), a fact which indicates that the velocity model succeeds in describing the geodynamical distortions of the test network since the official release of HGRS87. When the backdating extends to the pre-1987 period, the rms of the residuals continues to show a slight decrease and it tends to be stabilized around a minimum value. In all transformation scenarios this stabilization occurs within the period 1965-1973 where the rms remains practically constant. The middle year of this stable period (i.e. 1969.00) was conventionally selected as the "reference epoch" for the HGRS87 coordinates. Note that almost 98%

of the test stations belong to third and fourth-order networks, which were observed during the period 1962-1981 (Takos 1989).

4.4 Transformation parameters between HGRS87 and ITRF2008

A few important remarks should be given in relation to the estimated transformation parameters from the previous tests and the results shown in Figure 4. Firstly we notice that the rotation parameters contribute significantly to the optimal fit between HGRS87 and ITRF2008 at all considered epochs. The six-parameter model reduces the rms of the transformation residuals by 9-12 cm – compared to the three-parameter model – in a repetitive manner throughout the backdating period. This is indicative of the orientation differences between the two frames, which seem to cause systematic discrepancies of their horizontal coordinates at the decimeter level in our test network.

Secondly, the reduction rate of the transformation residuals by the seven-parameter model is smaller compared to the other models, and it practically fades out in the pre-1987 period. This reveals that the scale parameter is able to absorb (a part of) the *accumulated* geodynamical distortions in the static HGRS87 coordinates. Such a result is somewhat also reflected in Table 1, where we see that the estimated scale parameter drops considerably when the ITRF2008 coordinates are backdated from the observation epoch (2007.236) to the HGRS87 "reference epoch" (1969.00). Since the scale parameter affects the geodetic latitude and not the geodetic longitude (see appendix), the network distortion due to the north/south secular motions of the test stations is aliased into its estimated value, along with a part of other systematic errors that originally exist in the Hellenic geodetic reference frame.

Table 1. *Estimated transformation parameters between HGRS87 and ITRF2008. The reference epochs characterize the ITRF2008 coordinates that are used in the adjustment model of Eq. (4).*

Transformation epochs	t_x (m)	t_y (m)	t_z (m)	ε_x (")	ε_y (")	ε_z (")	δs
	three-parameter model						
2007.236	201.31	-74.40	-246.42	-	-	-	-
1969.00	198.54	-74.95	-248.70	-	-	-	-
	six-parameter model						
2007.236	385.71	-20.16	-486.81	-3.72	9.20	-0.78	-
1969.00	357.72	-17.73	-462.09	-3.33	8.15	-0.30	-
	seven-parameter model						
2007.236	430.11	-101.05	-493.43	-2.25	10.60	-3.08	525×10^{-6}
1969.00	365.22	-31.38	-463.23	-3.08	8.38	-0.69	89×10^{-6}

Lastly it is noticed that the scale parameter has a negligible contribution to the optimal fit between the underlying frames when the ITRF coordinate backdating approaches the HGRS87 “reference epoch”; see Figure 4. The rms of the seven-parameter transformation residuals at 2007.236 is smaller by about 7 cm compared to the six-parameter transformation residuals, yet both models have the same performance at 1969.00. This does not mean that the scale difference of the two frames becomes insignificant, but it simply suggests that the seven-parameter model tends to be an over-parameterized choice for describing their coordinate differences. It is actually expected due to the limited (non-global) coverage of the test network and the absence of any height information from the general transformation model in Eq. (4). The estimated parameters from the six and seven-parameter models are highly correlated, and they do not accurately reflect the exact magnitude of the datum differences (especially for the origin offset) between HGRS87 and ITRF2008. Nonetheless, all transformation parameters shown in Table 1 correspond to optimal fittings between the coordinate sets in our test network, and their resulting residuals carry useful information for the accuracy assessment of the Hellenic geodetic reference frame.

4.5 Assessment of the HGRS87 accuracy at national scale

The accuracy of HGRS87 is dictated by three error sources: (i) the quality of the geodetic measurements that were used for its development, (ii) the systematic errors caused by modeling approximations and other suboptimal procedures during the frame realization, and (iii) the accumulated distortions due to the inhomogeneous station motions throughout its multi-year realization phase (1962-1989) and after its official release in the late eighties. The individual quantification of these error sources is a difficult task that requires the decoupling of their corresponding effects from the available data. In the context of this study we only separate – in an approximate and average sense – the (secular part of the) geodynamical distortions via the backdating procedure that was described in previous sections. It is emphasized that this error source affects the HGRS87 accuracy not only at national scale but also over smaller “irregularity zones” of the Hellenic velocity field (e.g. fault zones). The identification of such areas and the analysis of the frame distortion within their boundaries, with regard also to past seismic activity and related co-/post-seismic deformations, are beyond the scope of the present paper. Herein, we simply focus on the external evaluation of HGRS87 based on its comparison with ITRF/GNSS-derived coordinates which are referenced to various epochs with the help of a geodetic velocity model. The large number of test stations and their uniform distribution across the Hellenic area ensure that our assessment will give not only a statistically reliable view of its accuracy level at national scale, but it will also reveal most of its problematic aspects in different regions. From the presented results in the previous section (see Figure 4) we have already seen that the average consistency between the HGRS87 and ITRF2008 coordinates at national scale worsens by an amount of 4-5 cm per decade (since 1987) as a result of the inhomogeneous station motions.

The adjusted residuals from the six-parameter transformation between the HGRS87 and ITRF2008 coordinates (at the epoch 1969.00) give a snapshot of the inherent quality of the Hellenic geodetic reference frame. The mean magnitude of the vectorized residuals is 38 cm while their rms reaches 45 cm and their maximum value goes up to 215 cm. Similar statistics are also obtained from the seven-parameter transformation between the same coordinate sets at the same epoch; see Table 2. These values reflect a mixture of random and systematic errors that exist in the HGRS87 coordinates, and they depict their expected accuracy at national scale. Note that our results are influenced by the errors in the ITRF2008 coordinates and the HGVM2013 velocity model, yet these factors are expected to have minor contribution to the error statistics given in Table 2.

Table 2. *Statistics of the vectorized transformation residuals between HGRS87 and ITRF2008. The reference epochs characterize the ITRF2008 coordinates that are used in the adjustment model of Eq. (4). All values are given in meters.*

Transformation epochs	min	max	mean	σ	*rms*
			six-parameter model		
2007.236	0.02	2.50	0.49	0.26	0.55
1969.00	0.01	2.15	0.38	0.25	0.45
			seven-parameter model		
2007.236	0.02	2.27	0.43	0.23	0.48
1969.00	0.00	2.13	0.38	0.24	0.45

As a large part of the transformation residuals reflect local systematic errors in the Hellenic frame of almost constant magnitude, the *relative accuracy* of HGRS87 is usually better and it can reach even a few cm for horizontal distances up to several kilometers. Consequently, the use of HGRS87 may still be sufficient for ordinary cadastral surveys, yet it cannot provide a reliable framework to support large-scale surveying and mapping projects with cm-level, or even dm-level, accuracy requirements.

The map shown on the top of Figure 5 depicts the residuals from the six-parameter transformation at the epoch 1969.00. Their behavior reveals that the HGRS87 frame is not characterized by uniform quality and it contains systematic errors with uneven spatial distribution. The residuals exhibit regional variations of several

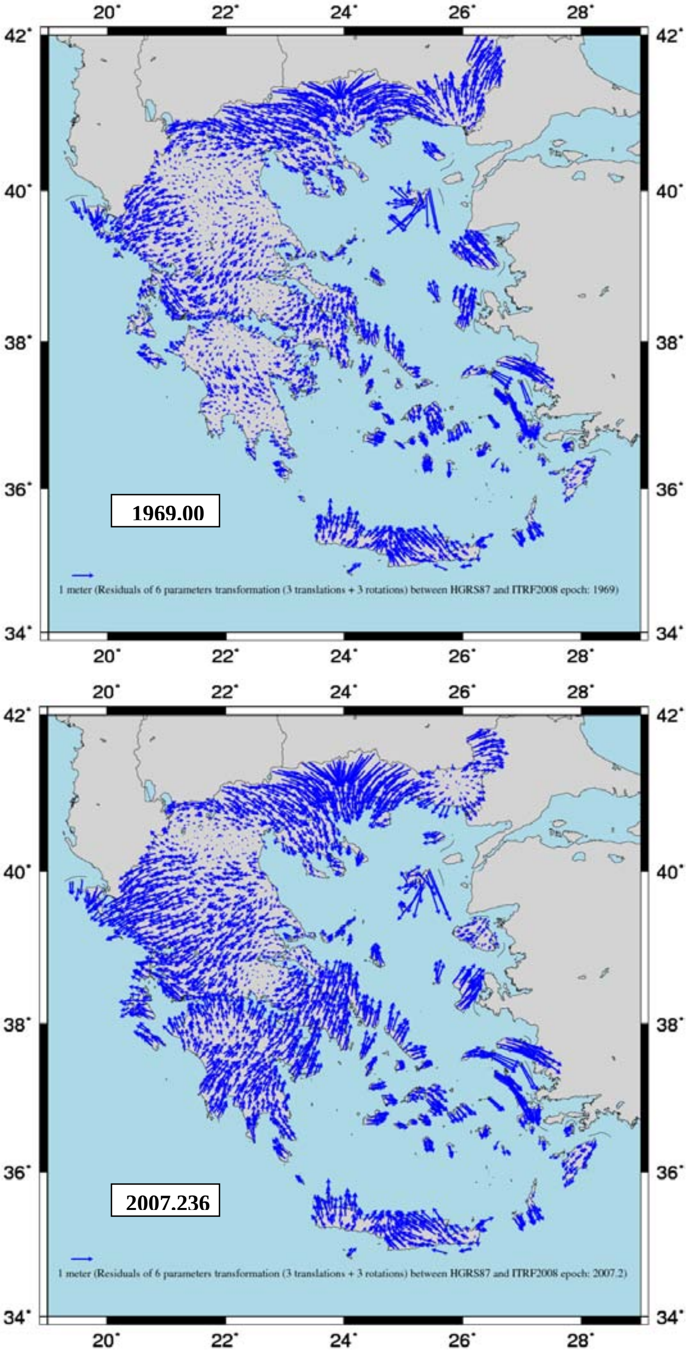

Figure 5. *Transformation residuals from the six-parameter model between HGRS87 and ITRF2008. The two maps correspond to different reference epochs for the ITRF2008 coordinates. The residual arrows shown in both maps are plotted with the same scale.*

decimeters while their orientation follows different patterns across the country, thus exposing the problematic aspects due to the fragmented formation of HGRS87 and the remaining geodynamical distortions from the different observation/realization epochs of the densification subnetworks. The weakest areas appear in the northern part (Macedonia, Thrace) and in most of the Aegean islands (Crete, Lesvos, Limnos, Chios, Kos, southern Evoia), a fact which corroborates with the analysis given in Takos (1989). Other scattered problematic areas are also seen in the western and some of the central parts of the test network. The smallest residuals occur in the southern part (Peloponissos) and most of the central mainland regions, where HGRS87 seems to be dominated mostly by random effects in the horizontal coordinates.

The comparison between the two maps shown in Figure 5 unveils the relevance of the ITRF2008 backdating procedure and the chronological degradation of HGRS87 due to the effect of the irregular velocity field. The latter adds up ~10 cm to the mean error budget of HGRS87 and brings its expected accuracy to the half-meter level at national scale (see Table 2). The regions that are mostly affected lie in southern and central Greece, and they show larger transformation residuals when the ITRF2008 coordinates refer to their observation epoch (2007.236) instead of the backdated epoch (1969.00). It is also evident that local systematic errors in HGRS87 can be partly "revoked" by the additive geodynamical distortions that have occurred in the test network. Characteristic examples of this situation are seen in Figure 5 for the area of Thrace in northeastern Greece and for the island of Lesvos.

The previous analysis was also performed using only the first and second-order BMs in our test network (48 stations). The results gave a similar picture to the one obtained by all available BMs (2431 stations), which is summarized below:

- The same "reference epoch" (1969.00) was empirically inferred for the static HGRS87 coordinates. Note that, according to Takos (1989), the terrestrial measurements in the first and second-order networks had been performed within the periods 1964-1986 and 1982-1989, respectively.
- The rms of the transformation residuals from the six-parameter model was 61 cm and 51 cm, at the epochs 2007.236 and 1969.00 respectively, showing again an average improvement of 10 cm due to the ITRF2008 backdating.
- The range of the transformation residuals is significantly smaller with maximum values going up to 1.2 m, compared to 2.15 m for the case where third and fourth-order BMs were used in the evaluation procedure.

All in all, the accuracy of HGRS87 at national scale seems to be worse than 40 cm regardless of the treatment of its systematic errors during the evaluation procedure (i.e. use of a scale factor in the transformation model, coordinate backdating via a velocity model, consideration only of higher-order stations). However, the

HGRS87 accuracy appears to vary significantly across Greece and it can reach higher level in certain regions, as explained in the next section.

5. Regional investigation of the HGRS87 accuracy

Hitherto the quality assessment of HGRS87 refers to national scale based on the residual analysis from frame transformations that were applied to the entire test network at various epochs. The presented results are influenced by the non-homogeneous horizontal motion of the network stations and they give an averaged view of the HGRS87 accuracy over Greece. To identify the regional quality characteristics of the Hellenic geodetic reference frame, we followed a similar procedure for the comparison of the HGRS87 and ITRF2008 coordinates in different parts of the test network, and the corresponding results will be summarized in the following sections.

5.1 Assessment of the HGRS87 accuracy in "stable" regions

For this evaluation the test network was divided into separate blocks, each of which is characterized by a rather uniform velocity field according to the HGVM2013 model. The selected blocks cover the following areas: (i) central/eastern Macedonia and Thrace, (ii) western Macedonia, Epirus and Thessaly, (iii) central Greece (includes the mainland part south of Thessaly and north of Peloponissos), (iv) Peloponissos, and (v) all the Aegean islands excluding Limnos and Lesvos. These are basically the same areas that were used in the remove-restore interpolation scheme during the development of HGVM2013, with the only difference being the treatment of Peloponissos as a single block for the purpose of our regional evaluation tests. In all cases the intra-block geodynamical distortion is expected to be minimal, and thus the comparison between the HGRS87 and ITRF2008 coordinates in each of these areas should give a more representative view of the HGRS87 accuracy and the magnitude of its internal errors.

The same evaluation methodology that was used in the previous section is adopted here: the six-parameter transformation model is fitted to the differences of the HGRS87 and ITRF2008 coordinates (as per Eq. (4)) using all available BMs within each block, and the resulting residuals are used to infer the regional accuracy of the former frame. Both sets of ITRF2008 coordinates, that is the original set at the observation epoch (2007.236) and the backdated set at the selected "reference epoch" of HGRS87 (1969.00), were employed for these fittings in every area. In Table 3 we give the per-block rms of the transformation residuals which reveal that the HGRS87 accuracy can be roughly classified into two levels across the country. Three out of five blocks, altogether covering the middle and western part of Greece, have rms values in the range of 21-25 cm while both of the other blocks appear to have almost doubled rms values (42-46 cm). This shows that the HGRS87 frame is much weaker and less accurate in the northeastern regions and in the Aegean islands, compared to the rest of the country.

Table 3. *Rms of the vectorized residuals from the six-parameter transformation between HGRS87 and ITRF2008 in "stable" parts of the Hellenic test network. The reference epochs characterize the ITRF2008 coordinates that are used in the adjustment model of Eq. (4). All values are given in meters.*

Test regions	1969.00	2007.236
central/eastern Macedonia and Thrace	0.46	0.44
western Macedonia, Epirus and Thessaly	0.21	0.22
central Greece	0.21	0.25
Peloponissos	0.23	0.22
Aegean islands	0.43	0.42

Overall, the magnitude of the transformation residuals remains almost the same in each block regardless of the used epoch for the ITRF2008 coordinates, a fact that is expected due to the "stable" character of the test areas. Note that the rms reduction by 4 cm in the case of central Greece (see Table 3) is probably caused by spatial irregularities in the velocity field of this area, which are effectively rectified by the backdating procedure. On the other hand, the slight rms increase of the transformation residuals in some blocks after the backdating should be attributed to the fact that the representative "reference epoch" of HGRS87 in these areas may be more recent than 1969.00.

5.2 Comparison of HGRS87 accuracy between the western and eastern part of Greece

The quality of HGRS87 seems to vary significantly between the western and eastern part of the country. This was already seen in Figure 5, and here it is confirmed by applying separate adjustments of the six-parameter transformation model in the western and eastern section of the test network w.r.t. the central meridian of λ=24°. The number of BMs within each network section is 1627 and 804, respectively. The analysis of the transformation residuals was performed separately for the epochs 1969.00 and 2007.236, and their resulting behavior is depicted by the cumulative histogram in Figure 6.

Firstly, we notice that the accumulated geodynamical distortions of HGRS87 have a more profound effect in the western section of the test network. Indeed, after the backdating, the rms of the transformation residuals is reduced by 12 cm in western

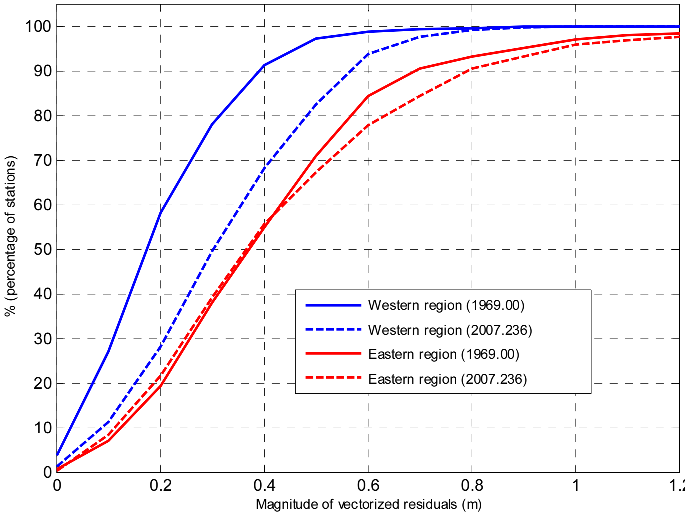

Figure 6. *Cumulative histogram of the vectorized residuals from the six-parameter transformation between HGRS87 and ITRF2008 in the western and eastern part of the test network. The reference epochs characterize the ITRF2008 coordinates that are used in the adjustment model of Eq. (4).*

Greece whereas the corresponding reduction for eastern Greece is only 3 cm; see Table 4. The percentage of stations having residuals smaller than 20 cm is almost doubled in the western section after the backdating procedure (reaching up to 60%) while the analogous percentage in the eastern section remains practically constant at 20%; see Figure 6. It is rather difficult to single out a reason for this difference between the two regions, which is probably caused by a convoluted "mixing" of the internal errors of HGRS87 with the accumulated geodynamical distortions in the eastern part of the test network.

Table 4. *Rms of the vectorized residuals from the six-parameter transformation between HGRS87 and ITRF2008 in the western and eastern section of the Hellenic test network. The reference epochs characterize the ITRF2008 coordinates that are used in the adjustment model of Eq. (4). All values are given in meters.*

Test regions	1969.00	2007.236
eastern Greece	0.54	0.57
western Greece	0.29	0.41

Secondly, it is evident from the results of Table 4 that the HGRS87 frame is more accurate in the western part than in the eastern part of Greece. This is further signified by the fact that the 90th percentiles of the transformation residuals at the epoch

1969.00 are 37 cm and 70 cm for the western and eastern section of the test network, respectively (see Figure 6). The average accuracy of HGRS87 in eastern Greece remains worse than half meter (1σ level) regardless of the epoch that is used for the comparison with the ITRF2008 frame. In western Greece, on the other hand, the average accuracy of HGRS87 is about 30 cm and worsens by an additional amount of 12 cm due to the accumulated geodynamical distortions up to the observation epoch 2007.236. These different accuracy levels should be attributed to a number of factors already mentioned in Takos (1989), including the larger number (and probably the higher quality) of geodetic observations that were used in western Greece during the HGRS87 realization, the weaker geometry of the national geodetic network in the eastern part of the country, and the larger systematic errors in the low-order eastern densification networks.

6. Conclusions

The analysis in this paper showed that the average accuracy of HGRS87 at national scale (or, more precisely, its consistency with a cm-level ITRF/GNSS-based national frame) is not better than 45 cm, even after removing the secular part of geodynamical distortions by a geodetic velocity model. Note that this value refers to the rms of the horizontal residuals at 2431 test points after a shift/rotation transformation has been applied between the underlying frames at a common epoch. The HGRS87 frame seems to be more accurate in the western part of Greece (~30 cm) compared to the eastern part (~55 cm) – the latter includes the areas of central and eastern Macedonia, Thrace and the Aegean islands. Furthermore, the following points should be stressed with regard to our evaluation results:

- As a result of the inhomogeneous velocity field in Greece, the HGRS87 looses about 4-5 cm every decade in terms of its expected accuracy at national scale.
- The aforementioned degradation is stronger in western Greece and weaker in the eastern part. This is evident by the fact that the coordinate backdating improves the fitting between HGRS87 and ITRF2008 by almost 30% in the western region, compared to only 5% in the eastern part of the country; see Table 4. The reason is probably the large internal systematic errors of HGRS87 that already exist in eastern Greece, which can “mask” most of the accumulated geodynamical distortions within this area.
- A scale parameter is able to absorb (a part of) the geodynamical distortions and it improves the average consistency between HGRS87 and ITRF2008 by about 7 cm, compared to the rigid transformation model, at the observation epoch 2007.236; see Table 2. Conversely, a scale parameter has negligible contribution when the frame comparison is performed at the HGRS87 “reference epoch” (1969.00) for the reasons discussed in previous sections.
- A part of systematic errors in HGRS87 is most likely aliased into the estimated

parameters of the transformation model(s) that were used in the evaluation tests. Therefore, the accuracy level that was inferred in this paper may be partially optimistic, both in national and regional scales.

As a final remark, we draw attention to the fact that it is not just the HGRS87 frame which has become outdated for geodetic and surveying applications in Greece. The official transformation model which is currently used for aligning GNSS surveys to HGRS87 will also gradually lose its claimed accuracy level (~10 cm, Kotsakis and Katsambalos 2008) due to the accumulating geodynamical distortions over the Hellenic area. This reveals the vital need to deploy a modern dynamic geodetic datum in Greece which will be able to support cm-level positioning accuracy at various spatial and time scales throughout the country.

References

Altamimi Z., Sillard P., Boucher C. (2002) ITRF2000: A new release of the International Terrestrial Reference Frame for earth science applications. *Journal of Geophysical Research*, 107, B2214.

Altamimi Z., Collilieux X., Métivier L. (2011) ITRF2008: an improved solution of the international terrestrial reference frame. *Journal of Geodesy*, 85, pp. 457–473.

Chatzinikos M. (2013) *Study of the earth's crust displacements in the area of Greece analyzing GNSS data*. PhD Thesis, School of Rural and Surveying Engineering, Aristotle University of Thessaloniki, Greece.

Chatzinikos M., Fotiou A., Pikridas C., Rossikopoulos D. (2015) The realization of a semi-dynamic datum in Greece including a new velocity model. *IAG Symposia Series*, vol. 143, doi 10.1007/1345_2015_93.

Delikaraoglou D. (2008) The Hellenic Positioning System (HEPOS) and its foreseeable implications on the spatial data infrastructure in Greece. *Technika Chronika*, Technical Chamber of Greece, 28(1-2), pp. 95-103.

Floyd M.A., Billiris H., Paradisis D., Veis G., Avallone A., Briole P., McClusky S., Nocquet J.-M., Palamartchouk K., Parsons B., England P.C. (2010) A new velocity field for Greece: Implications for the kinematics and dynamics of the Aegean. *Journal of Geophysical Research*, 115, B10403.

Gianniou M. (2008) HEPOS: Designing and implementing an RTK network. *GeoInformatics*, vol. 11, pp. 10-13.

Hollenstein C., Müller M.D., Geiger A., Kahle H.-G. (2008) Crustal motion and deformation in Greece from a decade of GPS measurements, 1993–2003. *Tectonophysics*, 449, pp. 17–40

Hellenic Mapping and Cadastre Agency – HMCA (1987) *The Hellenic Geodetic Reference System 1987*. Technical Report, Hellenic Ministry of Environment, Urban Planning and Public Works, Athens, Greece.

Katsambalos K., Kotsakis C., Gianniou M. (2010) Hellenic terrestrial reference system 2007 (HTRS07): a regional realization of ETRS89 over Greece in support of HEPOS. *Bullettin of Geodesy and Geomatics*, LXIX(2–3), pp. 151–164.

Kotsakis C., Katsambalos K. (2008) The official transformation model between the reference system of HEPOS and the Hellenic Geodetic Reference System of 1987. Presented at the symposium *HEPOS and modern geodetic reference systems: theory and practice, prospects and applications*, School of Rural and Surveying Engineering, Aristotle University of Thessaloniki, September 25-26, 2008.

Müller M.D., Geiger A., Kahle H.G., Veis G., Billiris H., Paradisis D., Felekis S. (2013) Velocity and deformation fields in the North Aegean domain, Greece, and implications for fault kinematics, derived from GPS data 1993-2009. *Tectonophysics*, 597/598, pp. 34-49.

Rapp R.H. (1993) *Geometric Geodesy (part II)*. Lecture Notes, Department of Geodetic Science and Surveying, Ohio State University, Columbus.

Soler T. (1976) *On differential transformations between Cartesian and curvilinear (geodetic) coordinates*. OSU Report No. 236, Department of Geodetic Science, Columbus, Ohio.

Torge W., Müller J. (2012) *Geodesy*. 4th edition, Berlin, de Gruyter.

Takos I.(1989) New adjustment of Greek geodetic networks. *Bulletin of the Hellenic Military Geographic Service*, 49(136), pp. 19–93.

Veis G. (1995) Reference systems and the realization of the Hellenic Geodetic Reference System 1987. *Technika Chronika*, Technical Chamber of Greece, 2nd special edition, pp. 16-22.

Vincenty T. (1985) On the meaning of geodetic orientation. *Bulletin Geodesique*, 59(2), pp. 189-199.

Appendix

The general transformation model that was used in our evaluation tests (see Eq. (4)) relies on the linearized expressions for the change in geodetic latitude and longitude due to origin, orientation and scale variation of the terrestrial reference system. These formulae are well documented in the geodetic literature (e.g. Soler 1976, Vincenty 1985, Rapp 1993) and they are provided herein for the sake of completeness.

Linearized similarity transformation of geodetic latitude

$$\varphi_i' - \varphi_i = \delta\varphi(t_x) + \delta\varphi(t_y) + \delta\varphi(t_z) + \delta\varphi(\varepsilon_x) + \delta\varphi(\varepsilon_y) + \delta\varphi(\varepsilon_z) + \delta\varphi(\delta s) \tag{5}$$

where φ_i and φ_i' are the geodetic latitudes of a terrestrial point w.r.t. different reference frames. The individual terms on the right side of the equation correspond to each of the seven transformation parameters between the underlying frames, and they are given by the following expressions:

$$\delta\varphi(t_x) = -\frac{\sin\varphi_i \cos\lambda_i}{M_i + h_i} t_x \tag{6}$$

$$\delta\varphi(t_y) = -\frac{\sin\varphi_i \sin\lambda_i}{M_i + h_i} t_y \tag{7}$$

$$\delta\varphi(t_z) = \frac{\cos\varphi_i}{M_i + h_i} t_z \tag{8}$$

$$\delta\varphi(\varepsilon_x) = -\sin\lambda_i \frac{aW_i + h_i}{M_i + h_i} \varepsilon_x \tag{9}$$

$$\delta\varphi(\varepsilon_y) = \cos\lambda_i \frac{aW_i + h_i}{M_i + h_i} \varepsilon_y \tag{10}$$

$$\delta\varphi(\varepsilon_z) = 0 \tag{11}$$

$$\delta\varphi(\delta s) = -\frac{N_i e^2 \sin\varphi_i \cos\varphi_i}{M_i + h_i} \delta s \tag{12}$$

where λ_i and h_i are the geodetic longitude and the geodetic height of the point w.r.t. the initial frame, a and e^2 are the semi-major axis and the squared eccentricity of the reference ellipsoid (which remain invariant in both frames), W_i is the auxiliary quantity:

$$W_i = (1 - e^2 \sin^2\varphi_i)^{1/2} \tag{13}$$

and M_i , N_i are the meridian and the prime vertical radii of curvature of the reference ellipsoid at the geodetic latitude of the given point, i.e.

$$M_i = \frac{a(1-e^2)}{W_i^3} \ , \quad N_i = \frac{a}{W_i} \tag{14}$$

Linearized similarity transformation of geodetic longitude

$$\lambda_i' - \lambda_i = \delta\lambda(t_x) + \delta\lambda(t_y) + \delta\lambda(t_z) + \delta\lambda(\varepsilon_x) + \delta\lambda(\varepsilon_y) + \delta\lambda(\varepsilon_z) + \delta\lambda(\delta s) \tag{15}$$

where λ_i and λ_i' are the geodetic longitudes of a terrestrial point w.r.t. different reference frames. The individual terms on the right side of the equation correspond to each of the seven transformation parameters between the underlying frames, and they are given by the following expressions:

$$\delta\lambda(t_x) = -\frac{\sin\lambda_i}{(N_i + h_i)\cos\varphi_i} t_x \tag{16}$$

$$\delta\lambda(t_y) = \frac{\cos\lambda_i}{(N_i + h_i)\cos\varphi_i} t_y \tag{17}$$

$$\delta\lambda(t_z) = 0 \tag{18}$$

$$\delta\lambda(\varepsilon_x) = \frac{N_i(1-e^2) + h_i}{N_i + h_i} \frac{\sin\varphi_i \cos\lambda_i}{\cos\varphi_i} \varepsilon_x \tag{19}$$

$$\delta\lambda(\varepsilon_y) = \frac{N_i(1-e^2) + h_i}{N_i + h_i} \frac{\sin\varphi_i \sin\lambda_i}{\cos\varphi_i} \varepsilon_y \tag{20}$$

$$\delta\lambda(\varepsilon_z) = -\varepsilon_z \tag{21}$$

$$\delta\lambda(\delta s) = 0 \tag{22}$$

where all auxiliary terms in the above equations have been previously explained.