Foreseeable Geometry Improvements from Future Global Navigation Flower Constellation Systems

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Abstract: This paper reports on recent results of simulations carried out for the purpose of evaluating the likely geometric advantages from a future Global Navigation Flower Constellation *(GNFC)* through comparisons of its performance with that of the conventionally designed constellations of *GPS* and *GALILEO* satellite systems. The evaluation of all three satellite systems is performed on the basis of the spatially varying Geometric Dilution of Precision *(GDOP)* scalar indicator which, at each earth location, reflects at a time instance the contribution of the satellite-receiver geometry which, in turn, affects the resulting accuracy of the navigation solution. The area of interest for the simulations was essentially global, although confined to the zone with latitude bounds -60 deg to +70 deg. The main design concepts and the advantages of the Flower Constellations with respect to the conventional *GNSS* orbits are discussed. Simulation results are presented in detail which shows that the tested GNFC constellation yields improved navigation performance in terms of *GDOP* values worldwide, except for a few small areas along the equator and mostly at oceanic regions where, by comparison, the current *GPS* and *GALILEO* constellations show slightly better geometric performance.

1. Introduction

The surveying, mapping and spatial information industry has benefited greatly from the use of Global Navigation Satellite Systems *(GNSS)*, involving a multitude of satellites, ground reference station infrastructure, various augmentations and state-of-the-art user equipment which allow determining accurate positions anywhere and anytime around the world. The US Global Positioning System *(GPS)* is currently the only such fully operational system which is being modernized by adding next generation satellites with more signal frequencies and codes, and with the aim to deliver to the users better accuracy, reliability and availability. Russia also operates *GLONASS*, its own *GNSS*, which is being revitalized as well. It is foreseen that fuelling further growth in the coming decade will be next generation *GNSS* systems, and their respective space augmentations, that are currently being studied for the purpose of improving further the future operational capabilities, especially with regard to observing individual sites or entire regions with optimal global coverage, while minimizing the number of satellites required in space in order to

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achieve such increased capabilities. It is within this context that, in this paper, using the concept of the *Flower Constellations*, we explore a key issue in the design of such constellations: the symmetry in space and/or in time distribution of the satellites which, in turn, affects the user-satellites relative geometry and subsequently the obtainable accuracy of a GNSS solution, and hence the anticipated Levels of Service towards the users.

1.1 Concept and applications of the Flower Constellations

The *Flower Constellation (FC)* concept was first introduced by Mortari et al. (2003) in an effort to generalize the design of circular inclined orbits having repeatable ground tracks. The basic characteristic of a Flower Constellation is that the orbits of all its satellites are compatible with a rotating reference frame. In other words “when viewed from an arbitrarily defined rotating reference frame, they all follow a single, identical closed-loop reference trajectory” (Wilkins, 2004). If the rotating frame is set to be an *Earth Centered Fixed (ECF)* frame (i.e. a frame rotating with the earth’s rotation rate), then the satellites will have perfectly repeated ground tracks. Due to the fact that the resulting orbit path of these satellites, as viewed from the rotating reference frame, resembles the shape of a flower petal, such a constellation set is suitably referred to as ‘Flower Constellation’.

In the original Flower Constellation theory, a *FC* is defined by eight parameters. Three are orbital parameters, which are identical for all the satellites belonging to the same *FC*: the perigee altitude (h_p), the orbit inclination (i), and the argument of perigee (ω). The other five are integer parameters playing a special role in defining the relative phasing of the satellites into a sequence of appropriate admissible positions: the number of satellites (N_s), the number of sidereal days required in order to repeat the ground track (N_d), the number of pedals (N_p) which equals the number of revolutions required to complete one period of repetition, and two integer parameters (F_n and F_d) which govern the phasing of the satellites, plus one extra phasing parameter (F_h) that was later introduced by Mortari and Wilkins (2008). Over the years, the original Flower Constellation theory has evolved into the so-called *2-D* and *3-D lattice theories*, as well as the most recent *necklace theory* (Mortari, 2012), which however are beyond the scope of this paper.

Satellite systems with orbital configurations based on the Flower Constellations concept have been proposed for a large number of applications, ranging from telecommunications and navigation to Earth and Deep Space missions. It has also been shown that *FCs* permit elliptical orbits, and can support just as well global, regional or spot areas services (e.g., reconnaissance and surveillance satellites, or close formation flying satellites). The fact that *FCs* are not confined to circular orbits and that they provide an unlimited range of satellite phasing configurations makes them a more efficient, yet complex, choice for a wider range of applications than those allowed by conventional constellations, such as the *GPS* and *GALILEO GNSS* systems which have been designed using another satellite constellations design concept known as the *Walker* formalism (Ruggieri et al, 2006) which is commonly used when seeking symmetrical distributions of satellites in an inertial frame of reference.

1.2 GDOP Definition

For the purposes of this study, in order to evaluate the *GNFC, GPS* and *GALILEO* constellations under consideration, we are going to use the well-known to GPS users *GDOP* metric which is dependent entirely on the geometry of the satellites within the view of a given location of interest. Using the expressions (1) and (2) below, *GDOP* is defined as the ratio of the square root of the trace of the covariance matrix ($C_{\hat{x}}$) for a GPS navigation (instantaneous positioning) solution divided by the accuracy of a single pseudorange measurement, expressed by the *User Equivalent Range Error* (σ_{UERE}) or the associated standard deviation of the actual measurements. Usually, for convenience and without compromising the resulting accuracy of the relevant computations, it is assumed that we are dealing with independent and identically distributed random range errors, i.e. the non-diagonal elements of $C_{\hat{x}}$ are zero (Yarlagadda et al, 2000; Langley, 1999).

$$C_{\hat{x}} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2 \end{bmatrix} . \tag{1}$$

$$GDOP = \frac{\sqrt{trace(C_{\hat{x}})}}{\sigma_{UERE}} = \frac{\sqrt{\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2 + \sigma_t{}^2}}{\sigma_{UERE}} \tag{2}$$

Small *GDOP* values indicate good satellite geometry which results in an instantaneous position having small uncertainty. When the *GDOP* values increase from epoch to epoch, it is indicative that the satellite geometry gets poorer and consequently the position fix uncertainty gets worse (Μπούσιας-Αλεξάκης, 2013).

2. Simulation considerations

For the current study we conducted various simulations with the aim to compare the geometric performance of a typical Flower Constellation for global navigation purposes, with the performance of the current GPS constellation and the currently implemented constellation of the GALILEO system satellites. The defining parameters and other key technical characteristics for these constellations, as used in the simulations, are briefly discussed below.

2.1 Global Navigation Flower Constellation

Park et al (2004) have suggested a *FC* for global navigation purposes, which has been named *Global Navigation Flower Constellation (GNFC)*. This particular orbit configuration design essentially consists of two separate *FCs*, each containing 15 satellites. The parameters defining each *FC* are presented in Table I.

Table I: *Flower Constellation Parameters (Park et al, 2004)*

FC Parameters	Values
N_p	2
N_d	1 Day
N_s	15
ω	180 deg
i	70 deg
h_p	20182 km

Table II: *GNFC Parameters: Ω & M in degrees (Park et al, 2004)*

Sat. #	Ω	M	Sat. #	Ω	M
1	0	0	2	24.0	311.99
3	48.0	263.99	4	72.0	215.98
5	96.0	167.97	6	120.0	119.96
7	144.0	71.96	8	168.0	23.95
9	192.0	335.94	10	216.0	287.94
11	240.0	239.93	12	264.0	191.92
13	288.0	143.92	14	312.0	95.91
15	336.0	47.90			

By using a symmetric phasing scheme (i.e. uniformly distributed satellites in the constellation's relative path) and assuming that the *mean anomaly at the initial time* (M_0) and the *right ascension of the ascending node* (*RAAN*) of the first satellite are zero, the data for all 15 satellites can be defined by the parameters presented in Table II. The two *FCs* have identical orbital elements and the only difference between them is that the argument of perigee is $\omega = 90$ *deg* for one of them and $\omega = 180$ *deg* for the other.

2.2 GPS Constellation

The GPS system was originally designed as a Walker 24/3/1 constellation, where

24 refers to the total number of satellites, 3 refers to the number of the orbital planes and 1 refers to the phasing parameter that defines the relative spacing between satellites in adjacent orbital planes. Although this pattern provided worldwide continuous coverage by at least 4 satellites at all times, it proved too sensitive to likely satellite failures. This eventually led to the system's re-designed constellation of 24 satellites in 6 equally spaced orbit planes, with nominal values for the semi-major axis, the orbital planes' inclination, and the eccentricity set to:
$a = 26{,}559.7\ km$, $i = 55\ deg$ and $e = 0$, respectively (DoD, 2008).

In 2011 this nominal constellation was modified so as to include three more slots in the core constellation and has since evolved into the so-called *24-expandable baseline GPS constellation*. In addition, spare satellites have been placed in orbit which are fully operational, but do not occupy a slot in the baseline constellation. In accordance with today's configuration, in our simulation scenarios we specified that at start time the GPS constellation consisted of 31 satellites.

2.3 GALILEO Constellation

Similarly, the *GALILEO* Constellation, in our simulations, was also designed a 27/3/1 *Walker* Constellation with an orbital inclination $i = 56\ deg$ and an orbital radius of 23,222 km. A spare operational satellite was also provided for each of the three orbital planes.

3. Simulation setups

3.1 Creating the scenarios- Inserting the Constellations

In order to carry out our simulations, we used the *Satellite Tool Kit (STK 9)* by Analytical Graphics Inc. *(AGI)*. We created three scenarios, one for each *GNSS* system constellation. The time period for each scenario was set to ten days, which is the same as the subtrack repetition period for the *GALILEO* satellites and also the longest subtrack repetition period amongst all three GNSS constellations.

The next step was to include in the simulations all three constellations in each of the three respective scenarios. We simulated the orbits of the 31 GPS satellites by using the *SGP4 Simplified Perturbations Propagator* and the *Two Line Elements* (*TLE*) orbital sets provided by the STK's satellite database. During the simulation period covered by each scenario and for the orbit propagation of each satellite, the *STK* would switch automatically between appropriate *TLE* sets in its database, so that orbital elements closer to the epochs of the simulated observations were always used. Similarly, we used *STK's Walker Tool* and inserted, as a 30/3/1 Walker constellation, the *GALILEO* satellites having an inclination $i = 56\ deg$ and an orbital altitude of 23,222 km. Finally, the *GNFC* constellation was included in the simulation by inserting all the satellites of the two sub-*FCs* specified in Tables I and II. In both cases, for the propagation of each *GNFC* orbit, we used the J4 (second-order) propagator which accounts for secular variations in the orbit elements due to Earth's oblateness, thus avoiding any likely perturbations which would otherwise slowly destroy the in-orbit compatibility of the *GNFC* satellites.

3.2 Definition and sampling of the Area of Interest

The area of interest for the current simulations was defined as the zone within the latitudes -60 deg to +70 deg, as this is the inhabited part of the globe where most human activities are taking place. A grid of 4212 points, separated in both latitude and longitude by a spacing of 3 deg at the equator, was created in order to analyze the time-varying *GDOP* values throughout the area of interest.

3.3 Computation of the Static GDOP values

The *GDOP* value for any point in space changes over time. Therefore, in order to evaluate and compare the quality of coverage that each constellation provides at each grid point, we have to use a representative static-like *GDOP* value, which covers the duration of each simulation scenario. Specifically, at each grid point over the duration of a scenario, the following variants of *GDOP* values were used: the average *GDOP* value *(aGDOP)*; the maximum *GDOP* value *(maxGDOP)*; and the upper bound limit *(upbGDOP)* that the *GDOP* values did not exceed for 90 percent of the times. Each of these static-like *GDOP* values for each grid point was computed from the time-varying values of *GDOP*, using a sampling time step of 300 sec. At each epoch, the corresponding *GDOP* value was always calculated using all visible satellites at that epoch that had an elevation angle greater than 10 deg. In addition, the minimum number of available satellites observed over the entire duration of a scenario was calculated and stored for each grid point.

3.4 Comparison Process

In order to compare the geometric performance of the *GNFC* to the corresponding performance for each of the *GPS* and *GALILEO* constellations, we first calculated the following differences between the respective static-like *GDOP* values produced by different satellite constellations at the i^th^ grid point:

$$\Delta GDOP_i' = GDOP_i^{GPS} - GDOP_i^{GNFC} \tag{3}$$

$$\Delta GDOP_i'' = GDOP_i^{GALILEO} - GDOP_i^{GNFC}. \tag{4}$$

In those expressions, for i = 1…4212, $GDOP_i^{GNFC}$ denotes the static-like *GDOP* value (e.g. *aGDOP, maxGDOP or upbGDOP*) which occurred for the *GNFC* constellation at the ith grid point, and $GDOP_i^{GPS}$ and $GDOP_i^{GALILEO}$ are defined in the same way for *GPS* and *GALILEO*. In that way, we obtained respectively: the

$a\Delta GDOP_i'$ and $a\Delta GDOP_i''$ indicators, when we used in (3) and (4), the average *GDOP* value at each point; the $max\Delta GDOP_i'$ and $max\Delta GDOP_i''$ indicators, when we used the maximum *GDOP* value; and the $upb\Delta GDOP_i'$ and $upb\Delta GDOP_i''$ indicators, when we used the upper bound limit that the *GDOP* values did not exceed for 90% of the times over the duration of a scenario. It should be noted that the term '90%below', is used for simplicity, in the tables and the discussions that follow, in order to denote the $upb\Delta GDOP_i'$ and $upb\Delta GDOP_i''$ indicators.

The calculated differences $\Delta GDOP_i'$ and $\Delta GDOP_i''$ are not transparent in understanding the amount of improvement or deterioration in the quality of the time-varying geometric coverage offered by each constellation. A more intuitive measure of performance is given by the expressions of relative percentage rate of change of the respective static-like *GDOP* values (e.g. *aGDOP, maxGDOP or upbGDOP*), between *GNFC* and each of the *GPS* or *GALILEO* constellations:

$$\%\Delta GDOP_i' = [\ (\ GDOP_i^{GPS} - GDOP_i^{GNFC}\ )\ /\ GDOP_i^{GPS}\ ]\ 100\% \quad (5)$$

$$\%\Delta GDOP_i'' = [\ (\ GDOP_i^{GALILEO} - GDOP_i^{GNFC}\ )\ /\ GDOP_i^{GALILEO}\ ]\ 100\% \quad (6)$$

It should be mentioned that all the necessary computations above, as well as the plotting for these quality indicators and their subsequent analyses were carried out by using various MATLAB scripts that were developed and worked within STK in a transparent way.

4. Results

4.1 Results for each Constellation independently

A simple statistical analysis has been applied in order to provide an overview of the geometric performance exhibited by each constellation. In Table III, the first three columns refer to the static-like *GDOP* values that were calculated for each grid point (average, maximum or 90%below), while the rows refer, for each GNSS system, to the typical statistics (average, minimum and maximum) obtained from the *GDOP* values of all 4212 grid points. For example, the element showing the value 1.71 in the first row for the GNFC system refers to the minimum observed value among the averaged *GDOP* values obtained at the 4212 points of the grid, from the scenario simulating the *GNFC* constellation. In addition to various static-like *GDOP* values, also shown are the statistics of the minimum number of visible satellites per grid point for each constellation, for the duration of each simulation scenario. From Table III, it is clear that for almost every indicator, the GNFC provides better results, in terms of the 'average', 'maximum' and '90%below' GDOP values, than those of the current GPS and the upcoming *GALILEO* systems. In order to inter-compare further the results between the *GNFC* and each of the *GPS* and *GALILEO* constellations, we have also computed the relative percentage differ-

Table III: *Statistical GDOP values for GNFC, GPS and GALILEO constellations.*

	Grid Stats	Average	Maximum	90% below	Min. Number of visible satellites
GNFC	Minimum	1.71	1.85	1.79	6.00
	Average	2.09	2.62	2.47	7.80
	Maximum	2.79	4.45	3.36	10.00
GPS	Minimum	1.88	2.44	2.11	4.00
	Average	2.14	4.66	2.71	6.20
	Maximum	2.41	18.43	3.64	8.00
GALILEO	Minimum	1.96	2.27	2.24	6.00
	Average	2.15	3.46	2.63	6.96
	Maximum	2.39	4.30	4.15	9.00

Table IV: *Comparison between GNFC and GPS (indicator $\%\Delta GDOP_i'$) as well as between GNFC and GALILEO (indicator $\%\Delta GDOP_i''$)*

	Grid Stats	Average	Maximum	90% Below
Comparing GNFC to GPS	Minimum	9.1	24.2	15.2
	Average	2.5	43.7	8.9
	Maximum	-15.6	75.8	7.8
Comparing GNFC to GALILEO	Minimum	12.88	18.78	20.32
	Average	2.69	24.14	6.09
	Maximum	-16.47	-3.46	19.23

ences expressed by (5) and (6) and the corresponding results are presented in the following section.

4.2 Comparing GNFC to GPS and GALILEO constellations

The statistical measures $\%\Delta GDOP_i'$ and $\%\Delta GDOP_i''$ resulting from the comparison between the *GNFC* and the *GPS* (and respectively the *GALILEO*) constellation are shown in Table IV. Positive numbers indicate *GNFC* improvements in *GDOP* values and, therefore, better satellite geometry and smaller position uncertainty. Evidently, for the majority of the results, the use of the *GNFC* leads to improved *GDOP* values. Only in the maximum value occurring amongst the averages of

GDOP indicators at all grid points, the *GNFC* presents worse results than *GPS* and *GALILEO*.

Figures 1 and 2 depict the percentile differences, i.e. the values computed using expression (5), between *GPS* and *GNFC* for the “90%below” (i.e., the $upb\Delta GDOP_i'$) and respectively the “average” (i.e., the $a\Delta GDOP_i'$) *GDOP* values.

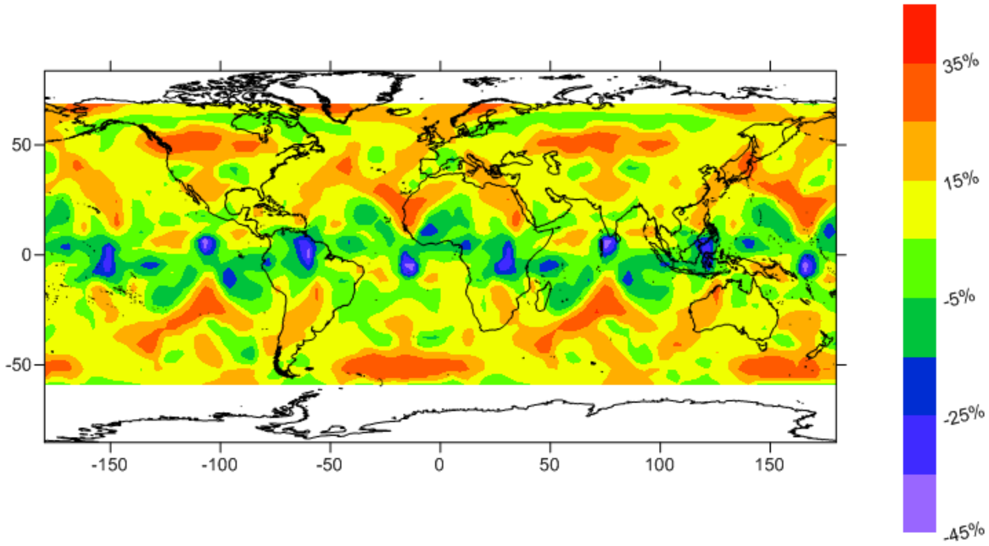

Figure 1: *Percentile Differences between GPS and GNFC for the "90% Below" GDOP values (i.e. equ. 5,* $upb\Delta GDOP_i'$*).*

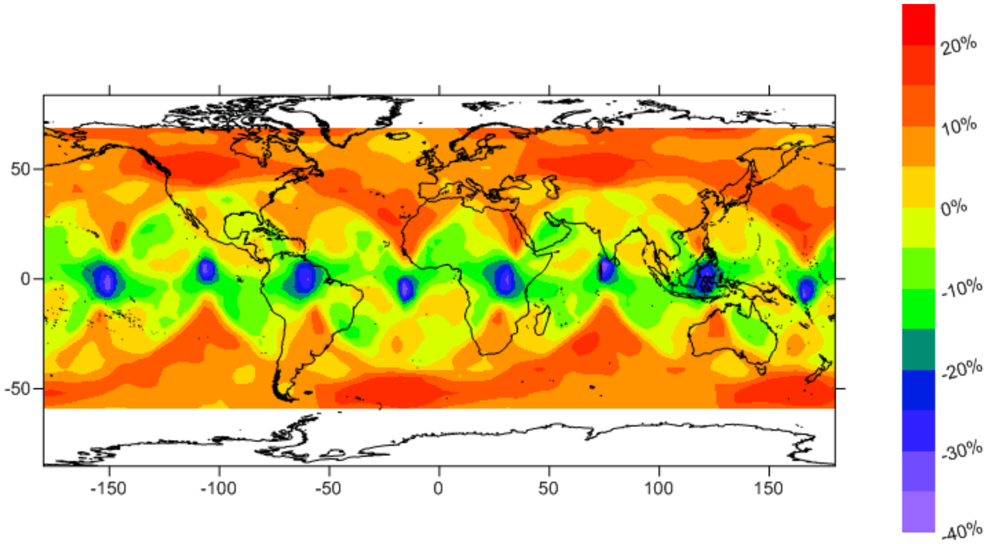

Figure 2: *Percentile Differences between GPS and GNFC for the “Average” GDOP values (i.e. equ. 5,* $a\Delta GDOP_i'$*).*

Similarly, Figures 3 and 4 depict the percentile differences, i.e. the values computed using expression (6), between *GALILEO* and *GNFC* for the "90%below" (i.e., the $upb\Delta GDOP_i''$) and respectively the "average" (i.e., the $a\Delta GDOP_i''$) *GDOP* values. In all four figures, the tones of grey which correspond to positive values illustrate areas where the *GNFC* presents better results than the *GPS* or *GALILEO* constellation. It is also noticeable that, in all cases, there are some "spotty" areas

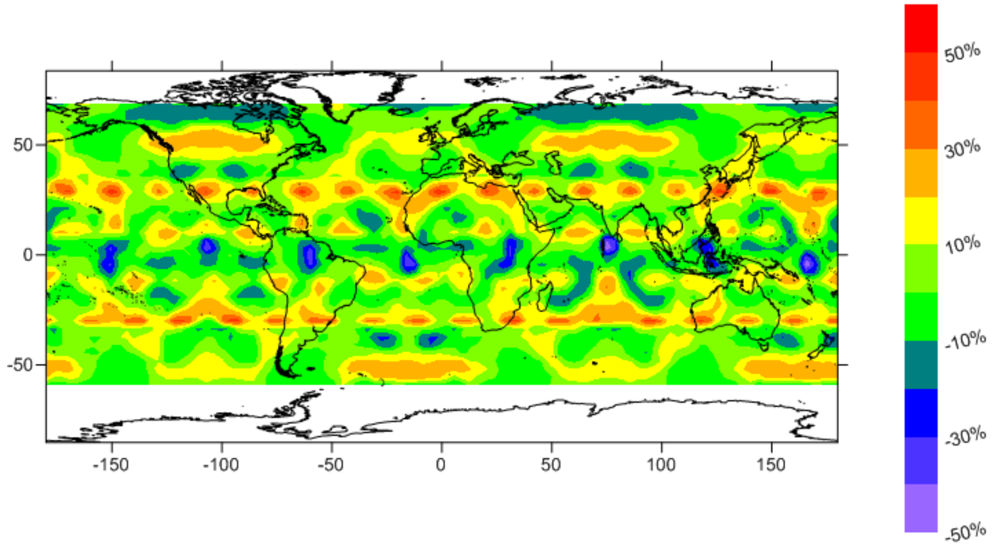

Figure 3: *Percentile Difference between GALILEO and GNFC for the "90% Below" GDOP values (i.e. equ. 6,* $upb\Delta GDOP_i''$*).*

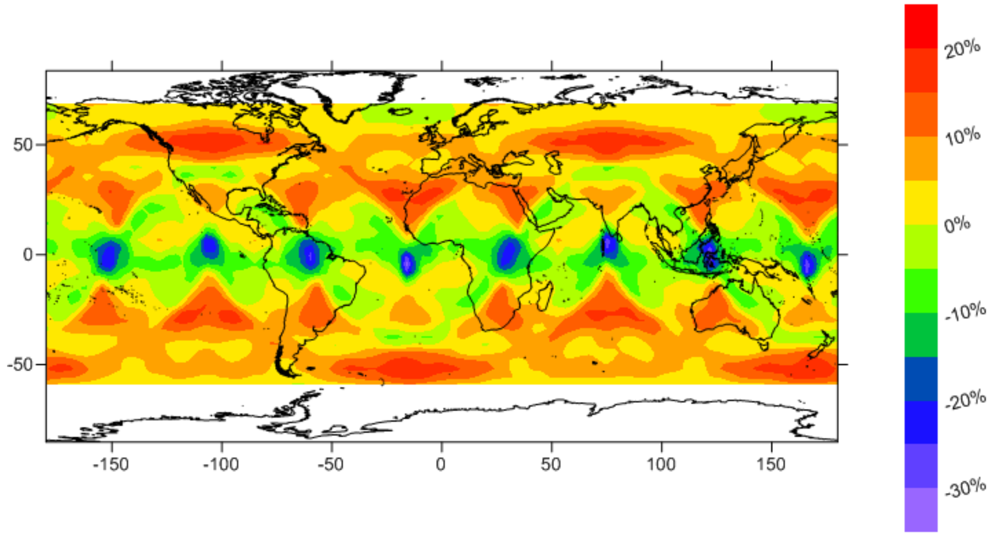

Figure 4: *Percentile Difference between GALILEO and GNFC for the "Average" GDOP values (i.e. equ. 6,* $a\Delta GDOP_i''$*).*

along the equator (indicated by the black color) where *GPS* (in figures 1 and 2) and respectively *GALILEO* (in figures 3 and 4) both exhibit better results.

5. Conclusion

In this study we have considered the problem of designing future *GNSS* systems and employed the Flower Constellations concept to evaluate an alternative *GNFC* design that, as supported by the results of our simulations, generally provides improved geometric performance, except in a few “spotty” small areas along the equator, where the current *GPS* and the upcoming *GALILEO* constellations exhibit slightly better geometric results. This is a characteristic of such *FC* designs that cannot be avoided. However, using a slightly more judicious choice of orbital parameters of the *FC* configuration, such areas can be placed in the least required regions (e.g. in oceanic areas).

One of the main complications that we were able to overcome in the present study was that the metrics by which a constellation is evaluated must be computed over time. Given the dynamic nature of this problem, this was accomplished by accounting for the orbital perturbations due to the Earth’s oblateness through the propagation of all satellites in a given constellation for some time of interest (the period for repeating ground-tracks), in order to compute the metric of interest in an extensive grid of points. Overall, this study has shown that future GNSS constellations may benefit from the above demonstrated results and other similar simulations planned as an extension of this study, in particular with regard to studying (and accounting for) the likely impacts of geometric performance in the Levels of Services to be provided by the future GNSS constellations.

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