Local and regional geoid modeling – Methodology and case studies

I.N. Tziavos[1], H. Denker[2], V.N. Grigoriadis[1]

[1] *Aristotle University of Thessaloniki, Department of Geodesy and Surveying, University Box 440, 54124, Thessaloniki, Greece*

[2] *Institut für Erdmessung, Leibniz Universität Hannover, Schneiderberg 50, D-30167, Hannover, Germany*

Abstract

High accuracy and resolution geoid models in different scales nowadays play a fundamental role in a wide range of the most demanding applications in geodesy and geosciences in general. The considerable improvement of these geoid models in recent years is mainly due to the progressive availability of satellite, airborne and terrestrial data sets. The purpose of this paper is first to briefly discuss the history of the European geoid (regional model) and the Hellenic geoid (local model) towards the data and the methods used as well as the accuracies achieved. Emphasis is also given to the methodological procedures, both in the space and frequency domain, primarily followed today for the determination of a high accuracy and resolution geoid model. In the second part of the paper the most recent gravimetric geoid model for Europe and a recent geoid solution for the Hellenic territory based on various combination schemes are presented and inter-compared. Moreover, these geoid solutions are compared (i) with corresponding heights derived from GPS/leveling at selected benchmark traverses on land and (ii) Sea Surface Heights (SSHs) derived from GPS buoy measurements at sea in order to validate and assess the consistency of the tested geoid models. An attempt is finally made towards the interpretation of systematic differences detected in the aforementioned comparisons and relevant conclusions are drawn regarding improved modeling and computation techniques.

Introduction

Local geoid determinations in Europe started already around 1900 based on deflections of the vertical (Helmert and Galle, 1914: Harz mountains, Germany) or torsion balance measurements (Eötvös 1906, 1909: Hungary). A first large-scale geoid solution for Central Europe was also based on deflections of the vertical and became available only in the 1940s (Wolf 1949). Until the 1980s, the emphasis was on the astrogeodetic technique. An example is the famous Bomford geoid (first version from 1954), which was continuously improved by including more data, and the last version finally utilized some 1000 deflections of the vertical (Levallois and

Monge 1978). Although a first gravimetric geoid calculation with mean isostatic anomalies was already carried out in 1949 by Tanni, the gravimetric technique received only little attention until the 1980s, when the GPS system came up and improved high resolution gravity field data sets as well as improved satellite gravity models became available.

The first high resolution gravimetric (quasi)geoid model for continental Europe (EGG1: European Gravimetric Geoid #1) was computed at the Institut für Erdmessung (IfE), Universität Hannover (now Leibniz Universität Hannover), Germany (Torge et al. 1982). The input data were $6' \times 10'$ and $1° \times 1°$ mean gravity anomalies as well as the global satellite gravity model GEM9; the absolute accuracy was estimated as roughly ±1 m, and the relative accuracy was at the level of several dm to about 1 m over distances of 100 km to 1000 km. The combination of the EGG1 model with about 5000 astrogeodetic deflections of the vertical led to the European Astro-Gravimetric Geoid EAGG1 (Brennecke et al. 1983) with improved accuracies in regions with sparse or erroneous gravity data.

The transition to geoid accuracies at the cm level succeeded for the first time in a local test area near Hannover by combining high resolution point gravity data (with a spacing of a few km), a digital terrain model and a global geopotential model (Denker and Wenzel 1987). In all, the 1980s brought major changes due to improved modeling techniques, the availability of high resolution gravity field data sets (e.g., point gravity data with a spacing of a few km), and significant advances in the computing power, allowing now regional geoid and quasigeoid calculations with accuracies improved by roughly one order of magnitude. On the other hand, also the accuracy demands substantially increased in the fields of geodesy, geophysics and engineering; especially the combination of GPS positioning techniques with classical leveling was – and still is – one of the main drivers for precise geoid and quasigeoid computations, requiring accuracies at the cm level.

At IfE, the experiences gained from the local geoid modeling in a small test area near Hannover were soon extended to larger areas, starting with Lower Saxony (federal state in Northern Germany), then continuing with entire Germany, and finally focusing on entire Europe (e.g., Denker and Torge 1993). The work on the determination of precise European geoid and quasigeoid models was supported by the International Association of Geodesy (IAG), from 1990 to 2003 within the framework of the International Geoid Commission, and since 2003 as an IAG Commission 2 project (CP2.1), denoted as the European Gravity and Geoid Project (EGGP). IfE acted as the computing center within these IAG enterprises, and a first major result was the European Gravimetric Geoid 1997 (EGG1997; Denker and Torge 1998). EGG1997 was based on the global geopotential model EGM96 (Lemoine et al. 1998) and high-resolution (point) gravity and terrain data available at that time. The evaluation of EGG1997 by GPS and leveling data revealed the existence of long wavelength errors at the level of 0.1 to 1 ppm, while the agreement over distances up to about 100 km was at the level of 0.01 – 0.02 m in many areas with a good quality and coverage of the input data (Denker and Torge 1998; Denker 1998).

However, since the development of EGG1997, significant new or improved data sets became available, including strongly improved global geopotential models from the CHAMP and GRACE satellite missions, new national and global terrain data sets, new or updated gravity data sets, improved altimetric results, as well as new GPS and levelling results. Last but not least, also the gravity field modelling techniques improved. Considering all these advancements, a complete re-computation of the European geoid and quasigeoid was considered appropriate and promised significantly improved accuracies, especially at long wavelengths. This task was pursued within the framework of the European Gravity and Geoid Project (EGGP) and led to interim results and status reports on a roughly annual basis (e.g., Denker et al. 2005, 2008, 2009). The last completely updated geoid and quasigeoid model for entire Europe is EGG2008 (Denker et al. 2008), which is based on the geopotential model EGM2008 (Pavlis et al. 2008) as well as completely updated high resolution gravity and terrain data for Europe. The evaluation of the EGG2008 model by independent GPS and levelling data (e.g., Denker et al. 2008, 2009) showed that the use of GRACE based geopotential models as well as upgraded gravity and terrain data led to significant improvements compared to EGG97 (in total by 25 % – 65 %). In addition, the long wavelength errors, existing in EGG97, were substantially reduced to typically below 0.1 ppm. The results indicated an accuracy potential of the gravimetric quasigeoid models in the order of 0.03 – 0.05 m at continental scales and 0.01 – 0.02 m over shorter distances up to a few 100 km, provided that high quality and resolution input data are available in the area of interest (Denker et al. 2009).

For the Hellenic territory, which constitutes the second test area of the present study, numerous geoid solutions were computed during the last 40 years with different resolutions and accuracies, employing various data combination schemes. The progress in the determination of the geoid in the area under study, either in terms of accuracy and resolution or with respect to the methodology and data availability, follows the development of the European geoid or quasigeoid as outlined above.

The first geoid solution for Greece was carried out by Balodimos (1972), who used observed astrogeodetic and estimated isostatic deflections of the vertical available along specific traverses on land. A first gravimetric geoid solution was computed later on by Arabelos (1980), where mean free-air gravity anomalies in blocks of $6' \times 10'$ and $1^{o} \times 1^{o}$ were combined using the Least Squares Collocation (LSC) method (Tscherning 1974). A geopotential model complete to degree and order 20 was employed as a reference field. The geoid heights were determined in a 0.5^{o} grid with an absolute accuracy in marine regions close to ±0.80 m after subtracting a 0.70 m bias. This accuracy estimate was achieved by comparing the gravimetric geoid heights with corresponding altimetric geoid heights computed on the basis of GEOS3 satellite altimetry. Arabelos et al. (1982) provided a new gravimetric geoid determination for the Hellenic area using additional gravity data, the global satellite gravity model GEM9 and the analytical integration of Stokes' formula. Different combined geoid solutions were carried out by Tziavos (1984, 1986, 1987, 1988), where astrogeodetic deflections of the vertical, 6′ × 10′ mean free-air gravity anomalies and geopotential models complete to degree and order 180 were used. The flexible LSC method in its stepwise form was employed for the optimal combination of the available heterogeneous data sets and the geoid heights were computed in a 0.5º grid. The above mentioned combined geoid models were based on empirical covariance functions of the data and their errors. The absolute accuracy of the geoid heights was estimated at the level of ±0.40 m and the relative accuracy was roughly between 0.20 m and 0.80 m over distances between 50 km and 200 km. Doufexopoulou (1985) presented another geoid determination for the area, mainly focusing on its geophysical and geodynamic correlations, and Fotiou et al (1988) investigated the correlation between the GRS80 combined geoid model (Tziavos 1984, 1988) and the upper crust density anomalies. Extended reviews on the different geoid models developed for the Hellenic area until the late nineteen eighties, comparisons between the different approximations and analytical evaluation tests can be found in individual investigations (see, e.g., Tziavos 1988, Arabelos and Tziavos 1989).

In the mid eighties the spectral methods, and especially the Fast Fourier Transform (FFT) techniques, were extensively used in gravity field modeling. The FFT techniques are basically applied to the evaluation of classical convolution integrals or series of physical geodesy (e.g., Stokes, Vening Meinesz, Molodensky) and present a number of advantages over other conventional analytical integral or collocation methods (see, e.g., Sideris 1984, Forsberg 1985, Haagmans et al. 1992, Tziavos 1993, 1996). These advantages are mainly related to the (a) high computational speed and efficiency using gridded data, (b) rapid computations over large areas due to the limited computational burden and (c) production of results on all grid points simultaneously by the elaboration of large volumes of high resolution data.

A first geoid solution for the Hellenic area by FFT was computed by Arabelos and Tziavos (1987) on a 6′ × 10′ grid along with the detection of systematic effects, seriously affecting the accuracy of the determined geoid model. A one-dimensional (1D) FFT-based high-resolution and high-accuracy geoid model for the Hellenic area using gravity and terrain data and the EGM96 geopotential model was computed by Tziavos and Andritsanos (1999). The solution was based on a new considerably improved free-air gravity anomaly database with a 5′ resolution both in latitude and longitude and a 1 km resolution DTM. The resulted geoid heights showed an absolute accuracy in marine areas at the level of ±16 cm in terms of standard deviation (std), based on comparisons with SSHs derived from TOPEX/Poseidon (T/P) satellite altimetry after taking into account a bias and tilt effect in order to remove existing datum inconsistencies. Comparisons of the computed geoid heights with corresponding heights from GPS/leveling on land showed a relative accuracy between ±5 cm and ±10 cm over distances of 30 km. This accuracy dropped to ±3 cm after the removal of a bias and tilt. Additionally, comparisons with the European geoid model EGG1997 showed an absolute agreement at the level of ±49 cm, but the largest differences were detected at the borders of the test area due to the low quality or lack of data in the original gravity database. It is evident from the validation tests of this geoid model that a significant improvement in the long wavelength part of the gravity spectrum was achieved mainly due to the use of the EGM96 geopotential model in the combined geoid solution.

Afterwards, several local geoid models were computed in different sub-regions within the wider area. These models were focused on geodetic, oceanographic, geodynamic, geophysical and engineering applications. It should be noticed that for the geoid model computed by Andritsanos (2000) with a 5′ resolution, where spatial and spectral methods were investigated, a frequency domain approach equivalent to LSC was developed employing system theory (Sideris 1996, Sansò and Sideris 1997). This spectral method was properly modified for the use of a system with multiple input gravity field related data and multiple output corresponding signals and resulted in the computation of geoid heights along with a Sea Surface Topography (SST) model for the Aegean Sea. After comparisons with SSHs derived from multi-mission satellite altimetry data, the absolute accuracy of the determined geoid was assessed at the level of ±8 cm. Local geoid models for different land parts of the wider test area (e.g., Paschalaki 2002, Andritsanos et al. 2004), with an accuracy ranging from one to several cm, showed the importance of geoid models in engineering and geodetic projects, where the combination of GPS and geometric leveling constitutes nowadays a common practice.

The experiences gained from the aforementioned geoid models for the Hellenic area and the requirements and goals posed by an international project sponsored by the European Union (GAVDOS project) led to the generation of a high resolution (1′ × 1′) and high accuracy gravity database (error estimate about ±3 mGal) in the southern part of the Hellenic area (Vergos et al. 2005). Based on the derived gravity database, a gravimetric geoid model was developed using the 1D FFT technique to evaluate Stokes' integral. Additionally, in the frame of the same project, altimetric geoid solutions, based on GEOSAT and ERS1 geodetic mission altimetry data, as well as combined solutions based on both gravity and altimetry data were determined. The accuracy of all geoid models was at the level of ±9 cm, as it was assessed by comparisons with stacked T/P SSHs, and the consistency between the models was found to be about ±2 cm. The use of this complete and homogeneous gravity data base was studied in a systematic way by Vergos (2006) for geodetic and oceanographic applications. In this study, different gravimetric, altimetric and combined geoid models were inter-compared and thoroughly investigated for the determination of the SST and velocities of currents in such a closed sea area as the southern Aegean Sea. An attempt was also made for the vertical datum unification in the Hellenic area by utilizing all available types of heights as well as tide gauge and mean sea level measurements. The implications of the geoid in geodynamic and geophysical interpretations were evaluated by Somieski (2008) for the northern Aegean Sea, where astrogeodetic geoid heights were correlated with Moho depths and isostatic anomalies in a tectonically and seismically interesting region characterized by strong gravity field signals.

The most recent geoid solution for the Hellenic area (HG2009) was computed by Grigoriadis (2009). This geoid model is based on a new free-air gravity anomaly database for the Hellenic area, which includes various datasets such as absolute gravity measurements, relative marine, land and airborne observations as well as marine free-air gravity anomalies derived from satellite altimetry. All these data sets were thoroughly tested for blunders and outliers using an advanced GIS methodology along with the classical processing techniques, taking also advantage of the global gravity field model EGM2008 and the new DTM and bathymetry (DBM) models (with resolutions varying from 100 m to 1 km) constructed in the frame of this study. The topography-bathymetry models were derived from corresponding national data sets and the international databases SRTM30 v2 (Farr et al. 2007) and SRTM30-plus v4 (Sandwell and Smith 2005). The finally derived gravity database presents a 2′ resolution both in latitude and longitude while its external accuracy was estimated at about ±2 to ±3 mGal. The corresponding geoid models computed for the Hellenic area using LSC and 1D FFT have the same resolution as the gravity database. The absolute and relative accuracies of the geoid models were estimated as ±1 to ±8 cm and 0.5 to 1 ppm over distances ranging from 5 km to 100 km, respectively. These accuracies were assessed by comparisons with corresponding GPS/leveling heights for land areas and SSHs and buoy measurements at sea. Recently, a new detailed geoid solution was completed, which is based on an upgraded version of the above mentioned gravity database and a thorough spectral analysis of the EGM2008 global gravity model and the heterogeneous data used in the combination scheme (Tziavos et al. 2010).

It should be noticed that after the geoid model from Andritsanos (2000) all relevant determinations included both geoid and quasigeoid computations, although in the aforementioned discussion, focusing on the history of the Hellenic geoid/quasigeoid, the term *geoid* is mainly used, which is not only for simplicity reasons but also due to the fact that the orthometric height / geoid height scheme is adopted in the Hellenic area.

It is evident that the accuracy of a geoid or quasigeoid solution, either in a local or regional scale, is directly linked with the quality of the available data and its uniform distribution in the area of interest. In this regard, the one cm geoid in absolute terms and at different scales is primarily expected from the improvement of the various national and international gravity field related databases, the updated terrain and bathymetry data sets and the refinement of airborne gravimetry and satellite altimetry measurements, mainly in the transition zones along the land/sea boundaries. Furthermore, significant advancements in the data processing are possible by exploiting recent and efficient geo-information tools which have become operational. On the other hand, the pure satellite geopotential models of the recent dedicated gravity field satellite missions CHAMP, GRACE and GOCE as well as the new geopotential model EGM2008 show a significant improvement in the long wavelength error budget in geoid modeling. All these heterogeneous and huge amounts of data can be optimally combined through efficient techniques in the spatial and frequency domain, also due to the nowadays available computational facilities. The combination methodologies and schemes can incorporate height data derived from GPS/GNSS or satellite altimetry (SSHs) not only for validation purposes but also for the unification of a reference system or national datum, which is another interesting motivation for the geoid modeling. Of course, it is worth mentioning that the 1 cm geoid can be achieved today in limited areas, where all the aforementioned data requirements are fulfilled and the computational and methodological tools can be easily accessed.

Methods

The most commonly technique used in geoid and quasigeoid modeling is the well known *remove-restore* technique which operates in two stages. First, the inherent gravity signal in the available measurements is separated into its frequency bands and then the geoid or quasigeoid signal is recovered and reconstructed in a stepwise mode. The methods discussed in the following are based on this fundamental procedure.

A distinction is necessary at this point between the terms *geoid* and *quasigeoid* and the corresponding terms *geoid height* and *height anomaly* used in the introductory section of this study as well as in the discussion given below. The geoid is the equipotential surface of the Earth which approximates the global mean sea level. On the continents this surface is generally located inside the terrain masses and as such it is not a harmonic function, as the anomalous potential T is not harmonic inside the topography. The geoid height N is related to T by Bruns' formula

$$N = \frac{T}{\gamma} \,, \tag{1}$$

but in this case T is the anomalous potential value at the geoid inside the topography (see, e.g., Forsberg 2005). The same formula as before holds for the height anomaly ζ, but now T is the potential value at the topographic surface or above it. The height anomalies evaluated at the topographic surface constitute the quasigeoid and Bruns' formula thus reads (e.g., Forsberg 2005)

$$\zeta(\phi,\lambda) = \frac{T(\phi,\lambda,r)}{\gamma} \ . \tag{2}$$

Depending on the height system used in a test area or country, the ellipsoidal height h of a GPS point can be expressed either in terms of orthometric height H and geoid height N or in terms of normal height H^* and height anomaly ζ according to the following equations:

$$h = H + N \ , \tag{3}$$

$$h = H^* + \zeta \ . \tag{4}$$

Therefore, the national height reference surface for a country using normal heights should be the quasigeoid, while in the case of orthometric heights the *geoid* is consistent. Both orthometric and normal heights involve the potential T, and based on a simple Bouguer topography model the conversion between geoid and quasigeoid is possible according to the formula (e.g., Forsberg 2005)

$$\zeta - N = H_P - H_P^* \approx -\frac{g_P - \gamma + 0.1967H}{\gamma} H = -\frac{\Delta g_B}{\gamma} H \ , \tag{5}$$

where Δg_B is the Bouguer anomaly, g_P is the gravity value at P and γ is the normal gravity. The interested reader should consult Heiskanen and Moritz (1967) and Forsberg (2005) for further details on the subject briefly commented here.

Spectral combination

The basic computation strategy is based on the remove-restore technique, considering high-resolution terrestrial gravity and terrain data in combination with a state-of-the-art global geopotential model (e.g., EGM2008, based on the GRACE satellite mission). In this procedure, residual observations are computed first by subtracting the effects associated with the global geopotential model and the digital terrain model (DTM – or more generally the mass model). The modeling techniques are then applied to the residual data. Finally, the effects of the global geopotential model and the DTM are added back to all predicted quantities. The general idea of the remove-restore technique is to use the global model for the recovery of long wavelength structures, the DTM for the modeling of the short wavelength components (in order to smooth the data and avoid aliasing effects), and the terrestrial gravity data for the computation of medium to short wavelength features of the gravity field. The remove-restore technique has become a standard procedure and was used successfully in connection with least squares collocation and integral formulas. Regarding the handling of the terrain effects, the residual terrain model (RTM) technique according to Forsberg and Tscherning (1981) is preferred as it only considers short wavelength signals; this is reached by treating only those masses between the actual topography surface and a smooth reference topography, e.g., computed by a moving average filter.

In this context, the primary gravity field quantity to be computed is the height anomaly or the quasigeoid undulation, with the advantage that only gravity field observations at the Earth's surface and in its exterior enter into the calculations, avoiding assumptions about the Earth's interior gravity field. A geoid model can then be derived by introducing a density hypothesis, which should be consistent with the one used for defining the corresponding orthometric heights (e.g., Helmert heights).

Regarding the modeling techniques, the exceptional long wavelength quality of the recent GRACE based geopotential models has to be considered, i.e., the long wavelength components of such models should be more or less adopted for the regional modeling, and the terrestrial gravity and terrain data should only contribute the medium and short wavelength components. This can be easily implemented using the spectral combination technique with integral formulas (e.g., Wenzel 1982). In connection with the remove-restore technique, the final height anomalies are obtained by

$$\zeta = \zeta_1 + \zeta_2 + \zeta_3 , \tag{6}$$

where ζ_1 is the contribution from the global geopotential model, ζ_2 is the effect of the terrain (mass) model, and ζ_3 is related to the terrestrial gravity data. Based on the spectral combination approach, ζ_3 can be computed by

$$\zeta_3 = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g_3 W(\psi) d\sigma , \tag{7}$$

where $\Delta g_3 = \Delta g - \Delta g_1 - \Delta g_2$ are the residual gravity anomalies (i.e., the free-air gravity anomalies Δg reduced for the effect of the global geopotential model Δg_1 and the terrain or mass model Δg_2), $W(\psi)$ is the integration kernel, R is the mean Earth radius, and γ is the normal gravity. The integration kernel $W(\psi)$ is defined as

$$W(\psi) = \sum_{l=2}^{\infty} \frac{2l+1}{l-1} w_l P_l(\cos\psi) , \tag{8}$$

where w_l are the spectral weights, and $P_l(\cos\psi)$ are the Legendre polynomials of degree l, depending on the spherical distance ψ. Compared to the well-known Stokes formula, the only difference is the spectral weight w_l, which determines how much signal is taken from the terrestrial gravity data at a certain degree l. The spectral weights can in principle be derived by deterministic or stochastic principles. In the least-squares spectral combination approach, the spectral weights depend on the error degree variances of the global geopotential model $\sigma_l^2(\varepsilon_1)$ and the terrestrial gravity data $\sigma_l^2(\varepsilon_{\Delta g})$:

$$w_l = \frac{\sigma_l^2(\varepsilon_1)}{\sigma_l^2(\varepsilon_1) + \sigma_l^2(\varepsilon_{\Delta g})}. \tag{9}$$

In the above equation the $\sigma_l^2(\varepsilon_{\Delta g})$ can be computed from the error covariance function of the terrestrial gravity data (e.g., Wenzel 1982), and the $\sigma_l^2(\varepsilon_1)$ can be derived from the coefficient standard deviations of the global geopotential model.

Fast Fourier Transform (FFT) methods

The FFT-based methods in spherical approximation are mainly used to evaluate Stokes' integral in the frequency domain towards the determination of geoid heights or height anomalies. Stokes' formula, similarly to the spectral combination method before, thus reads for height anomalies ζ_3

$$\zeta_3 = \frac{R}{4\pi\gamma} \iint_\sigma \Delta g_3 S(\psi) d\sigma, \tag{10}$$

where R is the mean Earth radius, Δg_3 are the residual gravity anomalies (similarly to eq. 7) and $S(\psi)$ is Stokes' function usually represented by an analytical summation of an infinite series. Since in applications of geoid and quasigeoid modeling the data are only available at discrete points, eq. (10) can be re-written analytically as

$$\zeta_3(\phi_P, \lambda_P) = \frac{R \Delta\phi \Delta\lambda}{4\pi\gamma} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \Delta g(\phi_n, \lambda_m) \cos\varphi_n S(\phi_P, \lambda_P, \phi_n, \lambda_m), \tag{11}$$

where $\Delta\phi$ and $\Delta\lambda$ denote the grid spacing of the data in latitude and longitude, respectively, and N, M define the block size and represent the number of meridians and parallels ($N \times M$ is the number of data points). By expressing the last equation as a convolution, ζ_3 can be evaluated at all grid points (data points) simultaneously by the two-dimensional (2D) FFT as follows

$$\zeta_3(\phi_P, \lambda_P) = \frac{R \Delta\phi \Delta\lambda}{4\pi\gamma} \mathbf{F}^{-1}\left[\mathbf{F}\left[\Delta g_3(\phi_n, \lambda_m)\cos\phi_n\right]\mathbf{F}\left[S(\phi_P, \lambda_P, \phi_n, \lambda_m)\right]\right], \tag{12}$$

where $\mathbf{F}$ denotes the 2D FFT operator and $\mathbf{F}^{-1}$ is the inverse 2D FFT operator. To account for the singularity of the kernel (Stokes' function) when the computation and the data point coincide, several techniques or approximation formulas were developed (see, e.g., Schwarz et al. 1990, Tziavos 1993). A new method for the evaluation of convolution integrals on the sphere by 1D FFT was introduced by Haagmans et al. (1992); the result is an exact evaluation of the integral on the sphere, which is achieved by applying 1D FFT for the convolution of the Stokes' kernel and the data in east-west direction combined with an integration (or summation) over parallels. The following formula is valid

$$\zeta_3(\phi_P,\lambda_P)=\frac{R\Delta\phi\Delta\lambda}{4\pi\gamma}\mathbf{F}^{-1}\left[\sum_{n=0}^{N-1}\mathbf{F}\left[\Delta g_3(\phi_n)\cos\phi_n\right]\mathbf{F}\left[S(\phi_P,\phi_n,\Delta\lambda)\right]\right], \tag{13}$$

where in this case $\mathbf{F}$ denotes the 1D FFT operator and $\mathbf{F}^{-1}$ is the inverse 1D FFT operator. The great advantage of the 1D FFT method is that the results are identical with those from numerical integration on the sphere, especially when it is optimally combined with the zero-padding technique towards the elimination of circular convolution effects (see, e.g., Tziavos 1993). Zero-padding concentrates on appending zeros around the original data set (usually at the 100% level) practically doubling the dimensions and consequently increasing the computational time, but this is not a real problem today. The 1D FFT method expressed by eq. (13) with discrete spectra both for the data and the kernel function was used for the local geoid computations in this study.

Least Squares Collocation (LSC)

Another approach for the determination of the ζ_3 term is LSC, a stochastic method widely used during the last decades not only for geoid and quasigeoid determination but also for other problems of physical geodesy. LSC is an adjustment, filtering and interpolation technique (Moritz 1980) that takes into account the statistical characteristics of the input data through the so-called auto- and cross-covariance functions between the different sets of observations. The great advantage of the method is the capability of approximating the anomalous potential T and its main components by combining different types of observables (gravity anomalies and disturbances, deflections of the vertical, different types of heights, etc.). On the other hand, it is worth mentioning that LSC is a time consuming method, even considering the nowadays available computational facilities, since it results in the solution of a linear system of equations, where the number of equations is equal to the number of the input data. For LSC, the ζ_3 term of the height anomaly is given by the formula

$$\boldsymbol{\zeta}_3=\mathbf{C}_{\hat{\zeta}\Delta\mathbf{g}}\left(\mathbf{C}_{\Delta\mathbf{g}\Delta\mathbf{g}}+\mathbf{C}_{\mathbf{ee}}\right)^{-1}\Delta\mathbf{g}_3\ , \tag{14}$$

where all bold printed symbols denote vectors or matrices. In eq. (14), $\boldsymbol{\zeta}_3$ is the set of height anomaly signals to be predicted, $\Delta\mathbf{g}_3$ is the residual gravity anomaly vector (observation; full gravity anomaly reduced for the effect of the global geopotential model and the topography as in the previous two methods), $\mathbf{C}_{\hat{\zeta}\Delta\mathbf{g}}$ is the cross-covariance matrix between the prediction signals and observations, $\mathbf{C}_{\Delta\mathbf{g}\Delta\mathbf{g}}$ is the auto-covariance matrix of observations (input data) and $\mathbf{C}_{\mathbf{ee}}$ is the error covariance matrix of the observations, usually a full diagonal symmetric matrix, of which the diagonal elements are the error variances of the observations. The error covariance matrix of the predicted height anomalies is given by the formula

$$\mathbf{E}_{\hat{\zeta}\hat{\zeta}} = \mathbf{C}_{\hat{\zeta}\hat{\zeta}} - \mathbf{C}_{\hat{\zeta}\Delta\mathbf{g}}\left(\mathbf{C}_{\Delta\mathbf{g}\Delta\mathbf{g}} + \mathbf{C}_{\mathbf{ee}}\right)^{-1}\mathbf{C}_{\Delta\mathbf{g}\hat{\zeta}} . \tag{15}$$

In the above classical type of LSC the minimization condition requires the square of the norm of the signal plus the variance of the noise (input data error) to be minimized. Nevertheless, in combination schemes, where SSHs or height anomalies derived from, e.g., GPS/leveling are incorporated into the model along with gravity anomalies, an alternative of the classical procedure is used and known as *parametric least-squares collocation* (PLSC) in the physical geodesy literature (see, e.g., Tscherning 2005). The parameters introduced in this advanced model can represent datum inconsistencies between the heterogeneous input data sets or other systematic effects existing in the observations. In the case of combining gravity anomalies with height anomalies (or geoid heights) derived from GPS/leveling measurements it is possible to determine simultaneously datum-shift parameters within a 4 or 7 parameter similarity transformation model. The minimization condition of PLSC requires additionally, comparing to the classical LSC, the square of the norm of the parameter vector to be minimized. The $\boldsymbol{\zeta}_3$ vectors of signals and parameters in PLSC can be estimated by

$$\boldsymbol{\zeta}_3 = \mathbf{C}_{\hat{\zeta}\Delta\mathbf{g}}\mathbf{C}^{-1}_{\Delta\mathbf{g}\Delta\mathbf{g}}\left(\Delta\mathbf{g} - \mathbf{AX}\right) , \tag{16}$$

$$\mathbf{X} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{C}^{-1}_{\Delta\mathbf{g}\Delta\mathbf{g}}\mathbf{A} + \mathbf{W}\right)^{-1}\left(\mathbf{A}^{\mathrm{T}}\mathbf{C}^{-1}_{\Delta\mathbf{g}\Delta\mathbf{g}}\Delta\mathbf{g}\right), \tag{17}$$

where $\mathbf{A}$ is a vector of partial derivatives (design matrix) with respect to the transformation model parameters, $\mathbf{X}$ is the vector of transformation parameters and $\mathbf{W}$ is an a-priori weight matrix (generally the zero matrix). The error covariance matrix of the predicted height anomalies is given in this case by

$$\begin{aligned}\mathbf{E}_{\hat{\zeta}\hat{\zeta}} &= \mathbf{C}_{\hat{\zeta}\hat{\zeta}} - \mathbf{C}_{\hat{\zeta}\Delta\mathbf{g}}\left(\mathbf{C}_{\Delta\mathbf{g}\Delta\mathbf{g}} + \mathbf{C}_{\mathbf{ee}}\right)^{-1}\mathbf{C}_{\Delta\mathbf{g}\hat{\zeta}} \\ &\quad + \mathbf{HA}\left[\mathbf{A}^{\mathrm{T}}\left(\mathbf{C}_{\Delta\mathbf{g}\Delta\mathbf{g}} + \mathbf{C}_{\mathbf{ee}}\right)^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\end{aligned} , \tag{18}$$

where $\mathbf{H} = \mathbf{C}_{\hat{\zeta}\Delta\mathbf{g}}\left(\mathbf{C}_{\Delta\mathbf{g}\Delta\mathbf{g}} + \mathbf{C}_{\mathbf{ee}}\right)^{-1}$.

Numerical results and discussion

Three quasigeoid models, briefly reviewed in the introduction of this paper, were selected and numerically tested:

(a) The last European gravimetric quasigeoid model EGG2008 (its boundaries are

$25.0^{o} \leq \varphi \leq 85.0^{o}$ and $-50.0^{o} \leq \lambda \leq 70.0^{o}$),

(b) the most recent and complete detailed quasigeoid solution for the Hellenic area HG2009 (its boundaries are $33.9^{o} \leq \varphi \leq 42.0^{o}$ and $18.9^{o} \leq \lambda \leq 29.8^{o}$) and

(c) the combined local quasigeoid model LHG2009 bounded in a restricted land region of northern Greece including also a minor sea part (its boundaries are $40.2^{o} \leq \varphi \leq 41.3^{o}$ and $22.4^{o} \leq \lambda \leq 23.8^{o}$; area of Thessaloniki).

The European quasigeoid model EGG2008 was computed by the spectral combination method, with the integral formula being evaluated by 1D FFT. The kernel function was based on the error characteristics of the terrestrial gravity data and the global geopotential model. Two versions of the HG2009 model were investigated, which were computed by 1D FFT and LSC in conjunction with the effect of the topography/bathymetry calculated by the classical terrain correction integral and the RTM method, respectively. In the last local model (LHG2009) the same gravity data as in case (b) were combined with GPS/leveling heights using the PLSC method. The basic computation strategy of all these quasigeoid determinations was the remove-restore technique, described already in the previous chapters, and the finally computed quasigeoid heights refer to GRS80. The main characteristics of the aforementioned quasigeoid models are summarized in Table 1 with respect to the type of input data used, the method followed to subtract the effect of the topography/bathymetry from the gravity anomalies, the reference geopotential model, the methodology followed in the computations and the resolution of the final model.

Table 1: *Main characteristics of the tested quasigeoid models.*

Geoid model	*Gravity data (Δg)*	*Effect of topography-bathymetry*	*Reference field*	*Method of computation*	*Geoid and quasigeoid heights*
EGG2008	$1' \times 1'$	RTM	EGM2008	Spectral combination 1D FFT	$1' \times 1'$
HG2009	$2' \times 2'$	TC/RTM	EGM1996	1D FFT	$2' \times 2'$
HG2009	$2' \times 2'$	TC/RTM	EGM1996	LSC	$2' \times 2'$
LHG2009	$2' \times 2'$	RTM	EGM1996	PLSC	$2' \times 2'$

In the European quasigeoid model EGG2008 about 5,5 million terrestrial gravity data (land, sea, and airborne) from more than 700 individual sources and about 13 million altimetric gravity anomalies were merged in order to compute a $1' \times 1'$ free-air gravity anomaly grid for the geoid and quasigeoid computations (Denker et al. 2008). The effect of the topography was calculated using the RTM technique ($15' \times 20'$ reference topography). The employed DTMs had resolutions ranging from $1''$ to $30''$ and involved several billions of elevation values. The utilized global geopotential model was EGM2008 (Pavlis et al. 2008) to degree and order 360.

In the geoid and quasigeoid solutions for the Hellenic area, more than ten thousands free-air gravity anomalies on land were used along with satellite altimetry and airborne gravity data as well as sea gravity data derived from the digitization of maps for the determination of the final 2′ × 2′ free-air gravity anomaly grid for the geoid and quasigeoid computations (Grigoriadis 2009). The effect of the terrain on gravity data was computed by the RTM technique and using the classical terrain correction (TC) integral. Different DTMs/DBMs were used for the computation of terrain effects for gravity anomalies with resolutions varying from 3″ to 30″. The results from the two methods were found to agree sufficiently, although a standard bias was detected when using the TC method to estimate the topographic effects for gravity anomalies. It is noticeable that the European gravity grid is of higher resolution than the corresponding Hellenic grid, which is related to the dense and homogeneous gravity coverage in the best surveyed parts of Europe, and regarding the Hellenic case to a lack of gravity data at the eastern borders of the area.

The statistics of the three tested quasigeoid models based on the above described gravity and height data are shown in Table 2. It should be noticed that the solutions for the Hellenic territory include the residual terrain model effects of land and bathymetry, whereas the European quasigeoid model only contains the effects of land elevations without a bathymetry model. Table 3 summarizes the statistical results of the differences derived from the comparisons between the aforementioned quasigeoid models.

Table 2: *Statistics of the quasigeoid models [unit: m].*

Quasigeoid model	*mean*	*std*	*min*	*max*
EGG2008	29.50	±11.51	0.27	46.13
HG2009 (FFT)	30.11	±11.59	0.72	46.75
HG2009 (LSC)	30.34	±11.53	0.95	47.06
LHG2008 (PLSC)	41.89	±1.19	39.07	44.11

Table 3: *Comparisons between the quasigeoid models [unit: m].*

Quasigeoid models compared	*mean*	*std*	*min*	*max*
EGG2008 - HG2009 (FFT)	0.62	±0.24	–0.18	1.64
EGG2008 - HG2009 (LSC)	0.85	±0.28	–0.11	1.80
EGG2008 - LHG2009 (PLSC)	–0.16	±0.46	–1.01	0.72
HG2009 (FFT) - HG2009 (LSC)	0.39	±0.14	–1.28	0.01
HG2009 (FFT) - LHG2009 (PLSC)	–0.72	±0.45	–1.51	0.21
HG2009 (LSC) - LHG2009 (PLSC)	–0.94	±0.44	–1.90	0.05

The European and the Hellenic quasigeoid models are in satisfactory agreement and the bias detected, expressed in terms of the mean value, can be attributed to the different reference fields used in the solutions as well as the different extents of the two test areas; the computation of the HG2009 model was carried out in an area located in the south-east part of the wider and higher resolution European gravity anomaly grid. Nevertheless, the bias between the EGG2008 model and the local combined solution (LHG2009) is considerably smaller. An explanation for this fact is that GPS/leveling heights were included in the LHG2009 solution and consequently the inherent systematic effects were partially reduced. The two versions of the Hellenic quasigeoid agree very well in the central part of the test area. The major disagreements were found in the eastern part due to the different behavior of the low quality gravity anomaly grid either incorporated into the frequency (FFT) or space domain (LSC) methodology. In figure 1 the HG2009 (FFT) model is shown, which evidently reflects several geodynamic features especially apparent in the southern part of the test area (e.g., Hellenic arc).

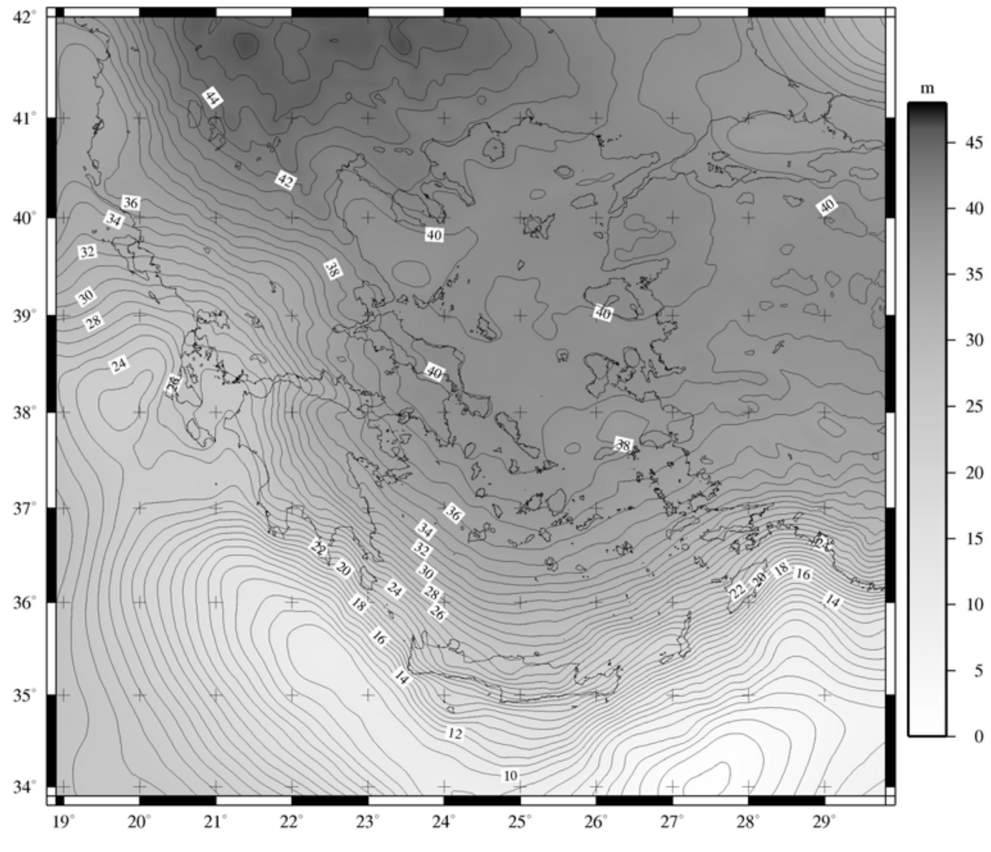

Figure 1: *The HG2009 (FFT) model (contour interval: 1 m).*

Table 4: *Comparisons between the quasigeoid heights from different models and corresponding heights from GPS/leveling [unit: m].*

Quasigeoid heights compared	*mean*	*std*	*min*	*max*
EGG2008 - GPS/Lev*	–0.43	±0.07	–0.58	–0.15
HG2009 (FFT) - GPS/Lev*	–0.80	±0.06	–0.97	–0.58
HG2009 (LSC) - GPS/Lev*	–1.06	±0.06	–1.22	–0.83
EGG2008 - GPS/Lev**	–0.42	±0.06	–0.50	–0.29
HG2009 (FFT) - GPS/Lev**	–0.79	±0.06	–0.88	–0.67
HG2009 (LSC) - GPS/Lev**	–1.06	±0.06	–1.13	–0.93
LHG2009 (PLSC) - GPS/Lev**	0.02	±0.05	–0.11	0.21

*84 control points **10 control points

In order to investigate the quality of the pure gravimetric quasigeoid models (EGG2008, HG2009) we used a number of 84 benchmarks from GPS/leveling in northern Greece (see Table 4). These heights were transformed to quasigeoid values in order to be directly comparable with the gravimetric solutions. The statistics of the differences between pure gravimetric and GPS/leveling quasigeoid heights at the control points are presented in Table 4. The above mentioned 84 benchmarks are located in the restricted test area of the combined solution LHG2009. For this reason, 74 GPS/leveling heights at benchmarks were included in the combined solution and 10 GPS/leveling heights kept out for an independent control. The comparison results of this local combined solution are given also in Table 4. The comparisons of the quasigeoid models with GPS/benchmarks not included in the solutions give an impression of the external accuracy of each quasigeoid approximation. Considering Table 4, significant improvements are realized in the recent gravimetric quasigeoid solutions for Europe and the Hellenic territory compared to previous geoid/quasigeoid models for both areas (see introduction). This fact reveals the major effect of the improved gravity databases used in the quasigeoid computations and the significant contribution of the recent geopotential models EGM1996 and EGM2008 regarding the minimization of long wavelength errors in geoid/quasigeoid modeling. The accuracy of the HG2009 model is better than that of the EGG2008 at the level of ±1 cm and even better is the corresponding accuracy of the local combined solution LHG2009 estimated at the level of ±1 cm in terms of standard deviation compared to HG2009 and ±2 cm compared to EGG2008. Figure 2 displays the LHG2009 (PLSC) geoid model along with the GPS/leveling benchmarks distributed in the area of Thessaloniki and belonging in the National Vertical Reference System. The distribution of the control points in a so restricted area is due to our intention the orthometric heights to belong in the same network and to be commonly adjusted.

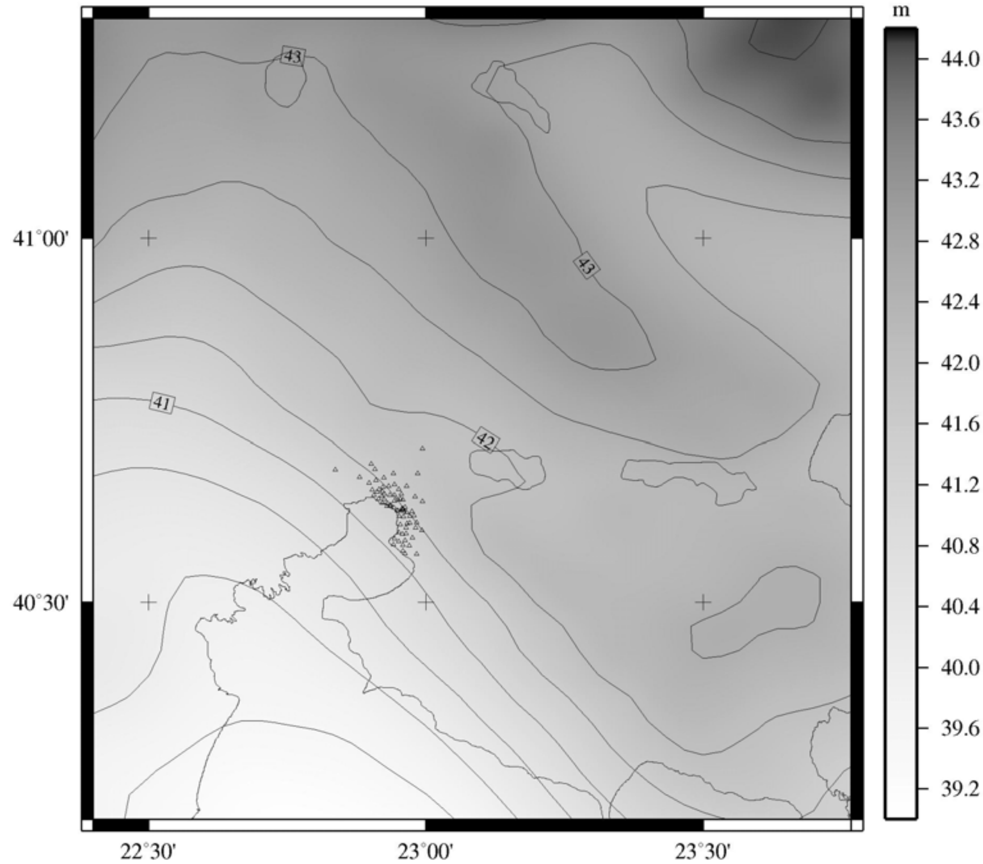

Figure 2: *The LHG2009 (PLSC) local geoid model in the central region of northern Greece (contour interval: 0.5 m). Triangles denote GPS/leveling benchmarks.*

The large disagreements observed with respect to the computed mean value (standard bias) between the gravimetric quasigeoid heights (EGG2008, HG2009) and the corresponding heights from GPS/leveling can be attributed to datum inconsistencies and further investigation is necessary in this direction especially for the Hellenic test area. This conclusion is also supported by the comparison results of the LHG2009 model, which shows only a minor bias value (2 cm) and is explained by the incorporation of GPS/leveling heights in the combined quasigeoid solution and the subsequent considerable reduction of systematic effects.

In a second validation test the geoid heights of the EGG2008 and HG2009 geoid solutions were compared with GPS boat and buoy measurements gathered during a dedicated measuring campaign for geoid determination in the north Aegean Sea in May 2005 (Müller et al. 2007). This campaign was realized in the frame of a joint project between the Geodesy and Geodynamics Laboratory (GGL) of Eidgenössische Technische Hochschule (ETH), Zurich, and the Department of Geodesy and Surveying of the Aristotle University of Thessaloniki, represented by the GeoGrav scientific group. Apart from the GPS measurements, astrogeodetic observations with the Zenith Camera DIADEM of ETH were carried out in order to determine highly precise deflections of the vertical. The measurement area is part of the north Aegean trough, which forms a continuation of the seismically active north Anatolian fault zone, a very interesting area from the geodynamic point of view. The kinematic positions of the buoys and the boat were determined through differential GPS carrier phase processing with respect to the reference stations. In order to derive the finally corrected SSHs and the SST from the instantaneous SSHs, a rigorous processing was carried out and several corrections were applied especially for tides and atmospheric effects (inverse barometer effect). The 20972 SSHs that resulted in this way were considered as geoid heights and they were directly compared with the corresponding geoid heights of the geoid models to be tested. This comparison was realized after the heights of the geoid models were interpolated for the positions of the SSH points along the marine GPS profiles. The statistical results of the differences are shown in Table 5.

Table 5: *Comparisons between geoid models and SSHs* derived from GPS buoy measurements [unit: m].*

Geoid heights compared	*mean*	*std*	*min*	*max*
EGG2008 - SSHs	–0.22	±0.07	–0.58	0.00
HG2009 (FFT) - SSHs	–1.54	±0.06	–1.80	–1.29
HG2009 (LSC) - SSHs	–1.20	±0.06	–1.43	–1.14

*20972 control points

Considering the calculated mean values of the differences in Table 5, it is evident that the regional European geoid model is less affected by systematic effects compared to the local solutions. Of course, and based on the comparison results mentioned before at GPS benchmarks on land, a corresponding marine geoid solution by combining gravity data and satellite altimetry derived SSHs could considerably reduce such systematic effects in a restricted sea area. The overall good performance of the local geoid solutions (±6 cm standard deviation of the differences) may be attributed to the improved and thoroughly tested new gravity database recently available for the Hellenic area (Grigoriadis 2009).

Concluding remarks and recommendations

The good performance of the regional and local geoid/quasigeoid solutions assessed by different validation tests, carried out either in the entire Hellenic area or in limited regions of interest, show the importance of high-resolution and accuracy geoid models for a wide spectrum of applications in geosciences and engineering projects. The thorough review of the geoid/quasigeoid solutions during the last two decades shows that the external accuracy of these solutions considerably improved, as the accuracy and resolution of the corresponding databases progressively improved as well. The substantial upgrades of the newly developed gravity database for the Hellenic area are due to the incorporation of new terrestrial, airborne and satellite gravity measurements and the methodological validation procedure followed in the unification of the database through advanced geodetic algorithms and GIS-based tools. A significant contribution to the creation of this accurate and homogeneous gravity database came also from the utilization of the recent high resolution DTMs/DBMs and global geopotential models (EGM1996, EGM2008, satellite models based on the CHAMP and GRACE missions). These advancements resulted in a reduction of the short/medium and long wavelength error budget in geoid modeling, respectively. Further improvements in the long wavelength band of the gravity spectrum towards the geoid modeling at local and regional scale is expected from the utilization of the data from the GOCE mission, which became available recently in the international geodetic community.

All the aforementioned advantages were considered also in the recent European geoid/quasigeoid model computation and led to substantial improvements in terms of absolute and relative accuracy. The use of the EGG2008 model in this study was twofold. First, an attempt was made to investigate the performance of a regional solution in a limited test area presenting dominant geodynamic features and a strong signature of the gravity field. Then, the regional geoid/quasigeoid model was employed to validate the corresponding local solutions and investigate existing systematic effects. These effects can be mainly attributed to the extent of the area of interest, the low quality of the height and gravity data sets incorporated into the initial versions of the corresponding databases, as well as the gravity data gaps. A significant reduction of the systematic effects, identified in the pure gravimetric quasigeoid model, was realized by a combined solution for a land sub-region of the entire Hellenic area, where a part of the available gravity database was used in conjunction with GPS/leveling heights at selected benchmarks. It is thus recommended to use this procedure for geoid/quasigeoid computations in extended test areas. In this regard and in marine applications, the gravity data can be optimally combined with SSHs derived from satellite altimetry and/or GPS buoy measurements.

Considering the overall good performance of the gravimetric geoid/quasigeoid solutions as described before and validated through numerical tests within this study, some additional conclusions and recommendations can be drawn with respect to geoid/quasigeoid applications at different scales. Geoid or quasigeoid heights determined with an accuracy of ±6 cm or notably better on land, fulfill today's requirements in a large number of applications in geodesy and can be used, e.g., for heighting via GPS and other engineering surveys. Moreover, accurate geoid heights in marine areas can be employed in the calibration of local or regional altimetric geoid solutions. The overall objective of the 1 cm geoid/quasigeoid determination can be achieved today for applications at local scales and in the frame of combined solutions, provided that the available height and gravity databases are of high quality and homogeneous coverage. This can be realized also in the Hellenic area within the proposed combined geoid solution scheme after a further densification and quality improvement of the available gravity database, the GPS leveling network on land and the GPS buoy measurements and satellite altimetry SSHs at sea.

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