The geodetic sciences in Byzantium

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Abstract: Many historians of science consider that *geodeasia*, a term used by Aristotle meaning "surveying", was not particularly flourishing in Byzantium. However, like "logistiki" (practical arithmetic), it has never ceased to be taught, not only at public universities and ecclesiastical schools, as well as by private tutors. Besides that these two fields had to do with problems of daily life, Byzantines considered them necessary prerequisite for someone who wished to study philosophy. So, they did not only confine themselves to copying and saving the ancient texts, but they also wrote new ones, where they were analyzing their empirical discoveries and their technological achievements. This is the subject of this paper, a retrospect of the numerous manuscripts of the Byzantine period that refer to the development of geodesy both in teaching and practices of surveying, as well as to matters relating to the views about the shape of the earth, the cartography, the positioning in travels and generally the sciences of mapping.

Keywords: Geodesy, geodesy in Byzantium, history of geodesy, history of surveying, history of mathematics.

Περίληψη: Πολλοί ιστορικοί των επιστημών θεωρούν ότι η γεωδαισία, όρος που χρησιμοποίησε ο Αριστοτέλης για να ορίσει την πρακτική γεωμετρία, την τοπογραφία, δεν είχε ιδιαίτερη άνθιση στο Βυζάντιο. Ωστόσο, όπως και η "λογιστική", δεν έπαψε ποτέ να διδάσκεται όχι μόνο στα κοσμικά πανεπιστήμια, αλλά και στις εκκλησιαστικές σχολές, καθώς επίσης και από ιδιώτες δασκάλους. Πέρα από το ότι οι δύο αυτοί κλάδοι είχαν να κάνουν με προβλήματα της καθημερινής ζωής των ανθρώπων, οι βυζαντινοί θεωρούσαν την διδασκαλία τους απαραίτητη προϋπόθεση ώστε να μπορεί κανείς να παρακολουθήσει μαθήματα φιλοσοφίας. Έτσι, δεν περιορίστηκαν μόνο στην αντιγραφή και τη διάσωση των σχετικών αρχαίων κειμένων, αλλά και στη συγγραφή νέων, όπου παρέθεταν τις εμπειρικές τους ανακαλύψεις και τα τεχνολογικά τους επιτεύγματα. Αυτό είναι και το αντικείμενο αυτής της εργασίας, μια ιστορική αναδρομή στα πολυάριθμα χειρόγραφα της βυζαντινής περιόδου που αναφέρονται στην εξέλιξη της γεωδαισίας τόσο στη διδασκαλία και τις πρακτικές της τοπογραφίας, όσο και στα θέματα τα σχετικά με τις απόψεις περί του σχήματος της γης, της χαρτογραφίας, του προσδιορισμού θέσης στα ταξίδια και γενικά στην εξέλιξη των επιστημών της αποτύπωσης.

Λέξεις κλειδιά: Γεωδαισία, γεωδαισία στο Βυζάντιο, ιστορία της γεωδαισίας, ιστορία της τοπογραφίας, ιστορία των μαθηματικών.

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1. Introduction

The Byzantine period began in 330 AD when Constantinople was inaugurated as the new capital of the Roman Empire at the site of Byzantion, the ancient colony of the Megarians founded in 658 BC by their king, Byzas. The division of the empire, already foreshadowed forty years earlier in the reforms of the Emperor Diocletian, was a realistic and intelligent response by the Emperor Constantine to major administrative problems, particularly to the fact that its eastern part could neither be governed nor defended effectively by a capital located so far west as Rome. Constantine himself had not aimed to divide central authority in the empire, but the reality of division quickly brought things to this point. Theodosius was the last emperor of a united eastern and western empire, and the final split occurred at the end of his reign in 395. Even in divided form, defense of the western empire was difficult and the emperor's position was precarious. Rome was plundered by the Visigoths and the Vandals, and finally in 476, with the proclamation of Odoacer as "king of all the tribes", the last Roman emperor Flavius Romulus Augustulus was deposed, inaugurating the "age of the barbarians".

From this period on, the western empire ceased to exist as "Roman". It was considered politically independent of Constantinople and was increasingly disconnected both in methods of governance as well as culture. The complete disappearance of Greek in the West during the 6th century meant that previous knowledge also disappeared, since education was largely a Greek monopoly of the ancient centers of Athens and Alexandria, which continued to flourish during the Early Byzantine period until their definitive closure in 529 by the Emperor Justinian. In contrast, in the West only some Latin handbooks managed to survive, with the exception of the most important Roman topographical work, the *Corpus Agrimensorum*, a collection of works dating from the 1st century BC to early 5th century AD by the official land surveyors of the Roman Empire, including Frontinus, Hyginus Urbicus, Siculus Flacus, Balbus, Nipsus, Apafroditus, Vitruvius, and Rufus. Written in the 6th or 7th century along the model of the works of Hero of Alexandria and today known as the Codex Arcerianus after its owner Giovanni Arcerio, it was for centuries one of the main tools of European civil servants.[1]

Due to the linguistic homogeneity imposed by the Greek language, in the eastern part of the empire there was a continuity in education independent of the tumult caused by various destructive wars and natural disasters, since the technological and theoretical work of the Greeks of the Classical and Hellenistic periods was readily available, given that it had been collected and classified in Alexandria. Despite this, however, the Byzantines - perhaps continuing the tradition of the Romans - did not distinguish themselves for their performance in theory, but for the

1 On Roman land surveyors, see Dilke (1992), Rossikopoulos (2006). On the Codex Arcerianus specifically, see Carden (1978).

practical use and application of scientific knowledge in everyday life. This was probably also due to the adoption of Christianity as the empire's official religion, since there training and education were reoriented towards theology. At least in the early years of Christianity, when the new religion clashed with the ancient pagan world, the study of the philosophers, and above all of mathematics, was neglected.

The four mathematical sciences - arithmetic, geometry, astronomy, and music - were included in educational curricula throughout the duration of the Byzantine Empire. This educational system of the four sciences, developed by Pythagoras, was preserved in Plato's Academy and in the Hellenistic Age as part of the *enkiklios pedia*[2], and subsequently in Byzantium as the "tetraktys". At the same time, there was no cessation in the teaching of logistic, essentially a form of elementary arithmetic for commercial purposes and geodesy, which was also considered a branch of arithmetic since in addition to teaching the demarcation of fields it also taught how to calculate areas and volumes by type and quality of crop.[3] The latter two subjects - logistic, geodesy - were also taught by private teachers outside schools, since they interested more students, given their practical application in daily life. Since people were not interested in their theoretical documentation, but only in their practical applications, mathematical types were employed mechanically without verification. Indeed, during the teaching of geodesy, *there was also a practical application involving measurements of various surfaces, that is, fields, vineyards and other such*.[4]

The mathematical *tetraktys* was the basis of mathematical studies not only in Byzantium but also in the West, to which it was introduced by the works of the 5th century Latin author Martianus Capella and the Roman official Boethius, the "last ancient Roman" philosopher, who also gave it the name *Quadrivium*, a term meaning "four roads." It would thus seem that mathematics played an important role in education, although the level of understanding was elementary. The goal of this educational program was to maintain knowledge, and to train traders, artisans, construction foremen, land surveyors and civil servants, rather than to serve as a springboard to new theoretical discoveries. The preserved manuscripts contain a number of collections with rules for plane and solid geometry for everyday needs

2 *Εγκύκλιος παιδεία*, it means "circular education", all the knowledge that needs to possess one. The curriculum.

3 Aristotle in his *Μετεωρολογικά* introduced the apt distinction between geometry and geodesy, and between arithmetic and logistic (i.e. "practical, applied" arithmetic). Geodesy (ge [γη] + deo [δαίω], to divide land), a term appearing for the first time, was according to Aristotle "practical geometry" i.e. modern-day surveying, in service to the needs of everyday life. *Μετεωρολογικά* (*Meteorology*; Latin: *Meteorologica* or *Meteora*) is a treatise by Aristotle, which contains his theories about the earth sciences (English translation by H. G. P. Lee, Loeb Edition, London 1978).

4 See Koukoules (1952).

that recall texts by Hero of Alexandria. The most popular works by this towering figure in Greek geometric discourse among the Byzantine authors of mathematical texts are: *Metrica*, which contains various measurement means for geometric shapes ranging from triangles to spheres as well as rules for calculating surface areas and volumes with numerical examples; *Definitiones* (*Ορισμοί*) and *Geometrica* (*Γεωμετρικά*), containing geometric measurements concerning practical applications; *Geodesia*, of which only a few fragments have survived, and *Stereometrica*, which discussed geometric measurements for solid objects. However, the work by Hero which formed a highly useful *vade-mecum* for the surveyors not only of Byzantium but of Europe until the 16th century was *On the Dioptra* (*Περί διόπτρας*), in the introduction to which it is noted that this was the first handbook of “dioptrics”, the science that dealt with the methods of practical geometry in combination through the use of the *dioptra*, an important instrument of the era comparable to the modern-day theodolite. According to the *Definitiones* of Hero, *What is the object of geodesy?*

> *It considers shapes that are not perfect and not clearly defined, ... like logistic (accounting, practical arithmetic). Thus, it measures heaps (of earth) as cones, and circular wells as cylinders, and sharp prisms as truncated cones. As geometry employs arithmetic, so this (sc. science, i.e. geodesy) employs logistic. It employs instruments - the dioptra for measuring fields, while it uses the rule, the plumb line, the gnomon and similar instruments for measuring distances and heights, sometimes with shadows, at others by sightings, and sometimes, it employs light reflection to solve these problems. As the geometer employs imaginary lines, so the geodesist measures perceptible lines. Of these, the most precise are determined by the sun’s rays, either through sightings or by the use of shadows, while the most perceptible are determined by tension and the drawing/pulling of ropes and the plumb line (level). Using these (instruments), the geodesist measures remote plots of land, the heights of mountains and walls, the width and depth of rivers, and similar things. Geodesy even divides (the earth), not only into equal parts, but also into secants and proportions and ratios, and often, in accordance with the value of fields.*

Concerning the distribution of inheritances and the settlement of disputes over property boundaries, we have references to the Theodosian Code, a collection of laws containing all the imperial decrees from 312, published at the order of Theodosius II (the Younger, 408-450) and included in the Justinian Code, a collection of imperial laws from the age of Hadrian to Justinian. There are also references to the *Pandects*[5], a collection of legal works and the most important work of Justinian

5 Also known as the *Digest* (Latin: *Digesta seu Pandectae*, adapted from Ancient Greek *πανδέκτης pandektes*, "all-containing").

legislation, compiled between 530 and 533. The writing of all the imperial decrees found in the Theodosian Code, as well as of all the later decrees of the 5th and 6th centuries was done in Latin, which remained the empire's official language during the 4th and 5th centuries. But at the university in Constantinople there was observed a decline in Latin and a preference for Greek, which was also the main language of the eastern empire and was supported by the School of Athens.

According to the Theodosian Code, the main arbitrators in settling disputes and making boundary determinations for any reason whatsoever were surveyors. If a disagreement broke out between landowners, the fee for the expert-surveyor was split equally by the disputants. In cases where boundaries needed to be re-established, e.g. after a flood, the surveyor's fee was paid by the public. The Justinian Code mentions that *when the head of the surveyors has completed two years [sc. of service], he acquires the humble office of "agens in rebus".* The *Digest* foresaw various punishments for surveyors who did not carry out their job properly, and there is also reference to the use of instruments. The relevant provision is entitled *If a surveyor declares an incorrect measurement (Si mensor falsum modum dixerit)*.

After the 4th century as well, "geodesists" were civil servants, military men, or free-lance professionals. Their duties also included land distribution for residential purposes. Although their job appears to have been very dignified, and they enjoyed public recognition, they were not included as a professional field among provisions obtaining for salaried work. Their fee was called an *honorarium*, i.e. money given in thanks and gratitude, and not *merces*, which meant "payment, wage". Perhaps this was because their work was defined more as a social service.[6] Their role in the daily life of Byzantium and the way they worked has been eloquently described by the distinguished Greek Byzantinologist Phaedon Koukoules[7]:

> *For determining the boundaries of a field and avoiding conflict between neighbors, the Byzantines customarily placed at its (viz., the field's) four edges distinguishing (land)marks which down through the centuries had various names, being called horoi, horia, synora, synoria, termonia, and orothesia. He (viz., the surveyor) was called upon to position these boundary markers. These consisted, as among the ancients, either of columns (stelai), or of sharp pointed stones, or of rocks lodged in the ground, or of ditches (tafrous) called traphous, or of mounds of earth, i.e. piles of earth or stones, or of trees or upright timbers which were called stavara, which was used as a synonym for cross (stavros).*

Since ancient times, the Greeks had delimited their fields with stones and boundary

6 See Cuomo (2001).

7 See Koukoules (1952).

markers (oria, called *ouroi* by Herodotus in the Ionian dialect) protected by the god Hermes. The word "Hermes" meant the spirit of the herma (*hermas, hermaion, chance find* or *hermaios lophos*: a pile of stones). From prehistoric times, such piles were built atop tombs, to indicate the separation of properties, as boundaries for lands and various regions, and at crossroads. Hence, we have the god's relationship with Hades, with ownership of property and with travelers. The text continues with a description of how the boundaries of fields are redrawn:

> *There these elders, called "boundary-indicators" in the papyri, and holding the Holy Gospel on which they have given an oath, measure by rope, if the boundaries have disappeared due to flood, indicate the (new) boundaries, at which men following after them position new boundary-markers.*

Apart from the above practical subjects of geometry, the teaching of geodesy also included learning about *the size of the earth, and the area of its uninhabited part and its inhabited part, which is reckoned to be one-fifth of the planet*.

2. Concerning the shape of the earth

The Church Fathers were of various opinions on the question of the earth's shape, depending on their philosophical and theological beliefs. All those who were closer to Platonic philosophy, including Origen Adamantius, the pioneer of Christian theology, Basil the Great, the 4th century's leading theologian, Ambrosius Mediolanum (known in English as Saint Ambrose), Saint Augustine of Hippo and John Philoponus accepted the idea that the earth was a sphere. On the other hand, all those Fathers from the region of Syria, who were also more inclined to follow the Old Testament literally, accepted the idea that the earth was flat. Diodorus of Tarsus, Photius Severian, Cosmas Indicopleustes, Saint John Chrysostom, and Saint Athanasius belonged to this latter tradition.

In the early 4th century, Diodorus of Tarsus supported that the earth was flat; according to the one and only mention by Photius Severian, Bishop of Gabala in Syria, Diodorus maintained that *the earth is flat and the sun does not pass beneath it at night; rather, it (the sun) travels to the northern regions, as if it were hiding behind a wall.* Towards the end of the same century, Saint John Chrysostom espoused the Old Testament view that the flat earth floated on waters gathered beneath the firmament. Saint Athanasius expressed similar views in his work *Against the Greeks* (*Κατά των Ελλήνων, Contra Gentes*), in which he turned his arrows against pagans. However, these preachings by the Church Fathers about the flat nature of the earth were a minority view for literate citizens from as early as the 3rd century BC and exerted no influence even on religious circles. For example, Saint John of Damascus (Ioannes Damascenus) offers the various views on the subject with taking a position himself, saying that this was not a matter of God's creation, but one for human research: *Some say that (the earth) was set and secured on water, like David; others that it (was set) on air.*

In the early 6th century, the two leading figures in the transmission of Hellenistic mathematics and astronomy in Constantinople were John Philoponus and Eutocius of Ascalon. Both men were students of the Neoplatonic philosopher Ammonius Hermiae, who reorganized the school of Alexandria after the terrible events that led to the death of Hypatia and defended the mathematics *Tetrabiblos* as a part of the curriculum in his dispute with Proclus many years after it was established. Eutocius was among the first important figures in the history of the transmission of the works of Archimedes and Apollonius. He wrote commentaries on various works by these two great mathematicians, dedicated to Anthemius of Tralles, the renowned architect of the Church of Aghia Sophia. The Monophysite John Philoponus was one of the greatest polymaths in the transitional period from Hellenistic to Byzantine science and the first Greek Christian to comment extensively on Aristotle. Unfortunately, all that has survived of his mathematical and astronomical work is a commentary to Nicomachus and a treatise on the astrolabe, a characteristic instrument in Byzantine times which was also used by the Arabs beginning in the 9th century, and subsequently in medieval Europe until the 17th century.

In his treatise, Philoponus avoided mathematical and theoretical documentation, addressing himself to those desirous of becoming familiar with and using the astrolabe. By the late 4th century, the astrolabe had already acquired the form it would retain for the next thousand years, and was equipped with a disk called the *mater* (*μήτρα*), one or more flat plates called *tympans* or *climate* (*τύμπανα* or *κλίματα*), and the *rete* (*αράχνη*, on account of its resemblance to the spider). Philiponus' treatise, which was divided into three parts (introduction, description of how to construct an astrolabe, and instructions for its use) provided a handbook appropriate for use in teaching in schools and for facilitating astronomical and astrological calculations. It was the earliest complete description of an astrolabe containing construction instructions. This work was probably based on notes from orally delivered lectures by Ammonius, themselves ultimately based on a lost treatise by Theon of Alexandria.

In his commentary on the first book of Aristotle's Meteorologica, Philoponus comments on the Aristotelian geocentric view of the universe, and the fact that the earth's size is small in relation to that of the sun and stars and the distances between them. Concerning the earth's circumference, he mentions that the ancients selected a section of the earth corresponding to an angle of one degree, as determined by the zenith directions at its ends, directed towards two stars this distance (viz., one degree) apart:

> *They also measured the earth's circumference. They took two stars one degree distant from one another, measured the projection of this distance on the earth, and found it equal to 500 stades. Considering that the largest cir-*

cle is divisible into 360 degrees, and multiplying it by 500 stades, they found that the earth's circumference is 180,000 stades.

Simplicius, a member of Plato's Academy in Athens and a student of the last great exponent of Neoplatonism, Proclus, presents similarities in his commentary on Aristotle's *On the Heavens*. He considers the center of the measuring instrument as the center of the world:

If someone looks at the earth with the entire heavenly sphere, there will in fact appear to be an unchanging relation with it (the earth) as fixed center. This is obvious from the fact that the centers of instruments used for astronomy - the observatory and astrolabes - which are placed in every part of the earth always define the center of the world.

He adds that the stars were identified with the help of the *dioptra*, and that the distance between them was measured with an odometer:

Since Aristotle referred to the measurement of the earth, observing that its circumference was said to be 400,000 stades, it would be good to record briefly - especially for those who do not believe in the genius of the ancients - the method of measurement they employed. They took a sighting with the dioptra of two fixed stars one degree distant from one another, i.e. 1/360th of the maximum circle of a sphere, determined with the dioptra the places where these two stars were at their zenith, and then measured the distance using an odometer. They found this distance to be 500 stades. Consequently, the circumference of the maximum circle of the earth is 180,000 stades, as Ptolemy calculated in his Geography.

Of course, neither man dealt with the measurement of the earth. Their references probably come from the commentary by the Peripatetic philosopher Alexander of Aphrodisias (2nd c. AD) on Aristotle's *On the Heavens* and *Meteorologica*.[8] Simplicius also mentions the low height of mountains in relation to the size of the earth:

In relation to a size as large as the earth's, the heights of mountains cannot reduce the spherical form of the earth or influence measurements that rely on the earth's curvature as their basis. With the aid of the dioptra, which measures [heights] from a distance, Eratosthenes demonstrated that the vertical length from the highest mountains to the lowest-lying ground is 10 stades.

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8 *On the Heavens* (*Περί ουρανού*; Latin: *De Caelo* or *De Caelo et Mundo*) is Aristotle's chief cosmological treatise, written in 350 BC, which contains his astronomical theory and his ideas on the concrete workings of the terrestrial world (English translation by W. K. C. Guthrie, Loeb Edition, London 1971).

Figure 1. *World picture from Christian Topography. The arched vault of heaven is represented above a flat, rectangular earth, where the sun is shown both rising and setting around the great mountain in the north. The firmament is at the meeting of the vault and the lower region. (11th century, Codex Sinaiticus graecus 1186, St. Katherine's monastery, Sinai).*

Philoponus makes a similar reference:

> *The highest points of mountains are above the clouds and the air. Even the tallest mountains do not rise much above the earth. The mechanics (engineering) experts who discover their heights with instruments normally employed for measuring mountains say that the tallest mountains have a height of 12 stades.*

Around the mid-5th century, Philoponus wrote his exegetical work, *Exegetika*, on the Creation of Moses (*De opificio Mundi, On the Creation of the World)* in his attempt to reconcile Aristotelian and Platonic cosmological views with the Biblical book of Genesis. In it, he endeavored to render the Ptolemaic system and spherical shape of the Earth acceptable to Christians.

Cosmas Indicopleustes, another contemporary of Justinian and perhaps slightly younger than Philoponus, fought Greek views of the Earth as round, and accepted that it was an elongated plain surrounded by the ocean, behind which was Paradise. His polemic was directed not only against pagans, but (and chiefly) against Monophysite Christians. As a merchant from Alexandria, he traveled for commercial purposes to Eastern Africa and Arabia, and got as far as the Indian subcontinent. Upon his return he became a monk at the monastery on Mt. Sinai, where he wrote *Christian Topography* (*Χριστιανική Τοπογραφία*, *Topografia Christiana*), a notable geographic work whose purpose was to found a new system of physical and

Figure 2. *After the installation of the Israelites at Shiloh, Joshua defines the land to be drawn to the seven tribes of Israel. Left: The draw of the land. Right: Going to measure the earth. Micrographs from The Octateuch of the Monastery of Vatopedi, Codex 602 (Ephesus Publications, 2005).*

mathematical geography in agreement with Christian teachings, as shown by the physical and astronomical interpretation of the Holy Scriptures. Despite his hostility to Greek geography and the Ptolemaic system, he incorporated into his work sundry elements and teachings of the ancient geographers, which makes it an important text in the history of science.

Towards the late 6th century, the level of applied mathematics and related instruments employed in Constantinople was fairly high, and both the Ptolemaic system and the Earth's spherical shape were by then taken for granted. It was not only the treatises of Eutocius and Philoponus that were assimilated but also the work by Anthemius of Tralles on concave mirrors, which followed the Alexandrian tradition of combining mathematics with engineering. In the fragments that have been preserved, he describes how it is possible to construct ellipses and parabolas from their tangents. The Emperor Justinian entrusted him with the rebuilding Aghia Sophia in Constantinople, which had been destroyed in 532 during the Nika revolt. This project was the greatest proof of the practical application of mathematical knowledge to technical topography and construction.

Returning to views concerning the Earth's shape and related works by Byzantines, in the 11th century we find the *Synopsis of Natural Science* or *On the nature of things concerning the utility of the heavenly bodies*[9] by the Greek official and translator of Arab texts Symeon Seth, written in similar fashion to the more interesting contemporary treatise by Michael Psellos, *De omnifaria doctrina*.[10] The latter's first book dealt with Earth, which lies at the center of the universe and is spherical

9 *Σύνοψις φυσικού* (Conspectus rerum naturalium) or *Σύνοψις φυσικής περί χρείας των ουρανίων σωμάτων* (De utilitate corporum caelestium).

10 *Διδασκαλία Παντοδαπή* (De omnifaria doctrina).

***Figure 3**. Measuring with rope (schoinismos, from The Octateuch of the Monastery of Vatopedi, Codex 602).*

in shape, with a circumference defined as 250,000 stades. Psellos had drawn this material largely from Aristotle's *Meteorologica*. Evidence mentioned as proof of the Earth's spherical shape included the changing times of sunrise, eclipses, ships which appear gradually as one approaches them, and the fact that in different regions of the earth different parts of the starry firmament are visible. The shape of the *oikoumene* (the inhabited part of earth) is elongated and occupies nearly the entire surface of the sphere. Problems of mathematical geography, the shape and extent of the inhabited part of earth interested scholars, clerics, and philosophers throughout the 1100+ years of Byzantium's life. By way of example, we may mention the poet and hymnographer Georgios Pisides, the cleric Nicephorus Blemmydes, and the philosopher and fervent supporter of Hellenism Georgios Gemistos or Plethon.

3. On measurements of land

Mathematics and engineering were significantly advanced by the Greek mathematician, engineer and architect Isidore of Miletus - who continued and completed the rebuilding of Aghia Sophia - and his students. Isidore published the works of Archimedes together with Eutocius' commentaries. And Isidore or one of his students compiled the 15th book of Euclid's *Elements*, invented a parabolic compass, and provided a commentary on Hero's *Kamarika (On arches),* which treated various stereometric and engineering problems of immediate architectural interest such as the construction of apses, etc.

In the mid-9th century, the leading intellectual figure in Byzantium, Leo the Mathematician, employed pairs of synchronized hydraulic clocks in an optical telegraph system he developed to convey messages from the frontiers of the empire to its capital. Evidence on Leo's youth informs us that he was born ca. 800 in Hypate, Thessaly, and that he received a fairly extensive education, although the university

***Figure 4**. Left: Quarrel during the measurement of the earth. The measuring rope is shown near the feet of the men. Right: The placement of a column as limit of the field. (From The Octateuch of the Monastery of Vatopedi, Codex 602).*

had closed in 726. He attended a grammar school in Constantinople and found a teacher of philosopher and mathematics on Andros, where there was also a library. He later established himself in the capital as a private tutor, offering a wide range of courses. When the Emperor Theophilos learned that the Caliph had invited Leo to Baghdad, he appointed him a state teacher to teach publicly in the Church of the Forty Martyrs. In 863, Caesar Bardas made him head of the new secular university in the palace of Magnaura, where Leo, as "*hypatos* of the philosophers" taught philosophy and the four mathematical sciences. Independent of any direct contribution to the mathematical sciences, his effort to rescue the work of the great classical mathematicians has ensured Leo an honorary place in the history of mathematics. Most of the manuscripts that linked Byzantine science with the ancient Greek tradition belong to his era.

In 938, we have a geodetic treatise whose title, *Geodesy*, was given to it later. Though presented as a work of Hero of Byzantium, it was in reality the work of an unknown author[11]. Its contents were drawn primarily from Hero of Alexandria's *On the Dioptra*, with elements related to calculation of areas and volumes drawn from the *Metrica*. The anonymous author described and stressed above all the military application of Hero's *dioptra* and of other surveying methods, essentially copying the first two paragraphs of Hero's *On the Dioptra*:

1. *Those who are considering a siege are incapable of measuring to calculate the heights of walls from a distance, or the intervals between distant points, or the widths of rivers. But those who are experienced in the use of charts and in observation with the dioptra can, by carrying out a survey, design siege towers that are the same height as walls, and bridges mounted on boats adapted to the*

11 Translated into Latin in 1572 (Barokius). The Ancient Greek text was published with a French translation by Vincent (1858). See also Sullivan (2000) and for Modern Greek translation, Rossikopoulos et al. (2005).

width of rivers, so that an army can cross the bridge or crossing unharmed and in formation.

This is followed by a large gap (*lacuna*) of around eight pages. They probably included, according to the references that follow in the text: a description of the *dioptra*, a section on basic arithmetic, another describing the measurement of a distance where one end is inaccessible, and an explanation of how to find the width of a river, probably offered as an application.

There follow problems whose solutions are mentioned as exercises, applications done in the Hippodrome of Constantinople:

2. *From a point from which a straight line can be sighted by ourselves, and which passes through this point at a right angle to the horizontal plane passing by our own position, without approaching the point. In other words: To measure the height of a wall from a distance.* (On the Dioptra, problem 12).
3. *Given two points at a great distance from one another, which are visible to the observer, find the horizontal distance between them.* (On the Dioptra, problem 10, 2nd solution).
4. *Another method of calculating the distance between two given points, in accordance with the diagram.* (On the Dioptra, problem 10, 3rd solution).
5. *Find the distance between two given points, as well as the position (orientation) of the straight line joining the points, without approaching either of them.* (On the Dioptra, problem 10, 1st solution).

There follows a problem involving the calculation of the areas of various shapes:
6. *Calculate the area of shapes enclosed by straight sections.* The techniques described refer to Hero of Alexandria's *Geometrica*, *Metrica*, and *On the Dioptra*, but the anonymous author also makes reference to Archimedes.

7. *Find the diameter of a circle, its circumference, and its area with the help of the dioptra, centered in the middle of the circle, without approaching the periphery.* To calculate the circumference, he multiplies the diameter by $3^1/_7$ and refers to Archimedes for calculating the area, which results if the diameter is multiplied by 1/4 of the circumference.
8. *Calculation of the volume of three-dimensional shapes: cubes, cylinders, cones, spheres and pyramids.* (The Pythagoreans, Archimedes, and Euclid's Elements are mentioned).
9. *Calculation of the volume of a water reservoir*. There is mention of the reservoir ("cistern") of Aetius in Constantinople, but the calculations are for the reservoir ("cistern") of Asparos, which was regular in shape.
10. *Calculation of the water supply from a spring.* (The corresponding problem and its solution are copied from Hero's *On the Dioptra*).

Following the structure of Hero's *On the Dioptra*, the anonymous Byzantine author refers to the measurement of the angular distance between two stars as an example of the use of the *dioptra* for astronomical purposes:

> *Now that we have referred to the terrestrial applications of the dioptra, we are ready - though we are on the ground - to attempt to observe the heavens, thanks to the possibilities of the dioptra. With it, we can determine the size of the sun and moon, and observe the distance between the stars, whether fixed or "wandering" (viz., planets). ... If we wish to observe a longitude or a distance along the zodiac, we do not perform a chance sighting, but (rather) we orient the dioptra parallel to the meridian; if we are looking for a north-south longitude or vice versa, we orient it to the equator.*

In the treatise, which is in fact a final spark of scientific and technological activity in mid-10th century Constantinople, applications of the *dioptra* continued in astronomy, but without any more references to its characteristics, or to terrestrial measurements. The texts of Hero of Byzantium also refer to the "lychnia" (λυχνία, lamp), a simple instrument (perhaps a streamlined form of *dioptra*) for measuring the height of an object.[12] The *lychnia* is also mentioned in the *Kestoi* ("Embroideries"), a work of miscellaneous content dating to the 3rd century by one of the first Christian Roman chroniclers, Julius Africanus.[13]

The work of St. Epiphanius, Bishop of Constanta and Archbishop of Cyprus, *On Measures and Weights*, which was composed in the 4th century, dealt with Biblical weights and measures. The Byzantine system of measurement for units of length was a development of the Greek, Hellenistic, and Roman system, but as concerned measures for surfaces, these were based on measurement units for volume, the preeminent one being the *modius*:[14]

> *During Byzantine times it is attested that the area of fields was measured and calculated by ropes, each rope having a length of ten to twelve orguias. But the area of fields was also calculated by plethra, which are repeatedly mentioned in books of agronomy and by Chrysostom, and even later by Eustathius of Thessaloniki, saying: plethron, a measure of land in circulation in many places even today. The Byzantines also calculated land areas*

12 The contents of the Geodesy were brought to Italy in the 12th century by the Italian mathematician and crusader Leonardo of Pisa (Fibonacci) in his book *Practica Geometriae*, which taught *inter alia* the determination of distances and heights with the use of similar triangles and an instrument called the geometric *gnomon*, a variant on Hero of Byzantium's *lychnia*.

13 This was a military handbook, which also made reference to practical geometry. Only one part of the work is preserved, in which through using the *dioptra* and Euclid's *Elements*, a solution is provided to two problems of interest to military topography: measuring the width of a river and measuring the height of a wall from a distance.

14 Koukoules (1952). See also Bantekas (1985).

from the total quantity of various measures of grain required to plant them. These were the measures normally referred to as the modius or modin – there are frequent references to "x-modia of land" or "fields of x-modii of wheat" - after these come the koilon or kabos and the choinix or choinikion. Finally, land area was measured according to yokes (of oxen), i.e. how many days it would require a yoke of oxen to cultivate and sow them.

Also mentioned among the practical measurements of various types of land are metrological passages preserved in several codices. The most important of these passages, whose authors are unknown, have the following titles[15]:

1. *Περί μέτρων (On measures).*
2. *Περί μέτρων γης (On land measures).*
3. *Περί αρούρας (On arable land).*
4. *Περί μοδισμού (On measurement using the modius).*
5. *Μέτρον γεωμετρικόν (Geometric measure).*
6. *Περί του μέτρου των αμπέλων του θέματος Θράκης (On the measure of vineyards in the theme of Thrace).*
7. *Έτερον περί των αμπελώνων της Θράκης και της Μακεδονίας (Another (work) on the vineyards of Thrace and Macedonia).*
8. *Περί μετροβολίας γης σκαφείσης και κυλισθείσης και σπορίμης και λιβαδιαίας (On the measurement of ploughed and turned over and arable and grazing land).*
9. *Άλλη μέθοδος του αμπέλου (Another method [sc. for measuring] vineyards).*
10. *Αμπέλιον των Θρακησίων. Του Οπτίματος. Του Οψικίου. (Vineyards among the Thracians; in Optima; in Opsikion).*
11. *Θέμα του Οψικίου. (On the theme of Opsikion)*
12. *Έτερον μέτρον του Οψικίου (Another [means of] measure of Opsikion).*
13. *Μέτρον αμπελώνων εις Νικομήδειαν και τον Κόλπον (Measurement of the vineyards in Nikomedia and Kolpos).*
14. *Έτερον μέτρον του Κόλπου (Another [means of] measure of Kolpos).*
15. *Έτερον μέτρον της Κίου του Καταβόλου και των Πυθίων (Another method of measurement of Kios and Katabolos and Pythia).*
16. *Των Πυθίων (On Pythia).*
17. *Περί αμπελοφύτων (On vineyards).*

15 These fragments have been published by E. Schilbach, *Byzantinische Metrologie* and A. Dain, *Metrologie byzantine Calcul de la superficie des terres*, with detailed information about the manuscripts containing them.

18. *Περί πλινθίων του μέτρου αυτών [των αμπελώνων] (On the measurement of rectangular plots of these [vineyards]).*
19. *Των πλέθρων το μέτρον (Measurement using plethra).*
20. *Μέτρον κυλίσματος (Measure of rough terrain).*

The next four texts concerning metrology of the earth (three by unknown authors, the fourth attributed to the mathematician Georgios Geometres and dated to the 14th century) are similar, though with references to methods or the definition of Geodesy, or to history and the beginning of geometry in Egypt:

1. *Μέθοδος της γεωμετρίας* (A method of geometry), starting from the famous text referred to in Egypt: *Καθώς ημάς ο παλαιός λόγος διδάσκει* (As for us, the word of old teaches …).
2. *Αρχή συν θεώ της γεωμετρίας* (The beginning of geometry with God's help), starting once again from matters concerning Egypt.
3. *Πως δε μετράς τόπον όσων μοδίων εστί* (How to measure how many modii [in area] a place is)
4. *Γεωμετρικόν Γεωργίου Γεωμέτρου περί γεωδαισίας* (Geometricon by Georgios Geometres (the Geometer) on geodesy), starting from the well-known definition: *Γεωδαισία εστίν επιστήμη* … (Geodesy is the science …).

All the above are mentioned as *Pseudo-Heronian texts*, and are modelled on Hero's works *Metrica*, *Stereometrica*, and *On the Dioptra*. Finally, there is a metrological text written in verse form attributed to Michael Psellos:

Του σοφωτάτου Ψελλού γεωμετρία δια στίχων (Geometry through verses by the eminently wise Psellos).

There is also a work incorrectly attributed to Michael Psellos on the four mathematical sciences, which was in fact written in 1008 according to a note at the end of the text. This work is divided into five parts:

1. *Συνοπτικόν σύνταγμα φιλοσοφίας* (Synopsis of philosophy).
2. *Ενταυ' αριθμών συντομοτέρα φράσις* (Here, a more concise version of numbers).
3. *Της μουσικής σύνοψις ηκριβωμένη* (A precise synopsis of music).
4. *Σύνοψις αύτη γεωμετρίας* λόγων (A synopsis of discourses on geometry).
5. *Αθροισις ευσύνοπτος αστρονομίας* (A concise summary of astronomy).

and is considered both anonymous and spurious, having been published under Psellos' name in 1532. It is attributed to Gregorios the Monk by some and by others to Romanos of Seleucia.

In addition to the above, an 11th-century manuscript preserves Hero's *Metrica*, the well-known collections known as *Geometrica* and *Stereometrica*, and similar works by Didymos and Diophanes. Another manuscript of the same period contained Hero's *On the Dioptra*.[16] A book dating to 1252 called the *Principle(s) of the great Indian Calculation* (*Αρχή της μεγάλης και ινδικής ψηφιφορίας*) contained a collection of an 11th-century collection of problems in arithmetic as well as Hero's *Definitiones*.[17]

During the second half of the 11th century, the Land Cadaster of Thebes, an example of a Byzantine cadastral code written by the empire's economic services, was drafted. Apart from cadastral tables, it preserves no diagrams; its importance lies in the fact that it provides valuable information about the Byzantine taxation system and agrarian economy in the 10th and 11th centuries. Three other documents, also from the northern Greek area and dating to the second half of the 11th century, present similarities with the Land Cadaster of Thebes.

John Pediasimos, a student of George Akropolites, lived in Achrida (Ochrid) and worked as an archivist for the Bulgarian church. He summed up the limited geometrical knowledge of his time in his *Geometria (by the eminently wise archivist of Bulgaria, Ioannes Pediasmos, a summary concerning the measurement and partitioning of land)*.[18] His sources, apart from Euclid and Hero of Alexandria, also included the *Geodesy* of Hero of Byzantium, which was widespread at the time.[19]

Around 1300, another student of George Akropolites, George Pachymeres, wrote the *Quadrivium (by the eminently wise George Pachymeres, a corpus of the four courses of arithmetic, music, geometry and astronomy),* which offers us a clear idea of the level of teaching of mathematics during the final period of the Byzantine Empire's cultural apogee.[20]

Already in the early 14th century, geodesy and applied arithmetic were no longer taught in schools of higher learning. However, the numerous manuscripts of this period demonstrate a heightened interest in these two applied sciences, given that they deal with problems people encountered in everyday life. With regard to geodesy, the Byzantine age concluded in the 15th century with a book on arithmetic that

16 *Περί διόπτρας*, included in the supplementary Codex Parisinus 607.

17 This was the first work to employ western Arab numerals. It has been handed down in the supplementary Codex Parisinus 387, and was the original reference book for Maximus Planudes for the writing of his *Ψηφιφορία κατ' Ινδούς, η λεγόμενη μεγάλη (Great Calculation according to the Indians)*.

18 Γεωμετρία (του σοφωτάτου χαρτοφύλακος Βουλγαρίας κυρίου Ιωάννου του Πεδιασίμου σύνοψις περί μετρήσεως και μερισμού γης).

19 See Friedlein, 1866, Schilbach, 1982 and Kaltsikis, 2005.

20 *Τετράβιβλος (του σοφωτάτου κυρίου Γεωργίου Παχυμέρη σύνταγμα των τεσσάρων μαθημάτων αριθμητικής, μουσικής, γεωμετρίας και αστρονομίας)*. Preserved in Codex 38 of the Biblioteca Angelia in Rome, published in 1940 by the publishing house of P. Tannery.

concerned subjects involving calculations, measurements, and planimetry (plane geometry), with collections of problems from the entire Greek past.[21]

4. Maps, astrolabes, and periploes

It was Maximus Planudes who towards the end of the 13th century revived interest in the geography of Ptolemy. A monk-scholar in the Monastery of Chora, a writer, an exponent of the spirit of the Palaeologan renaissance in Classical Greek, and a superb Latinist, he showed an interest in the translation of Classical works of Latin literature into Greek. Relying on a book entitled *Principles of the great Indian Calculation* (*Αρχή της μεγάλης και ινδικής ψηφιφορίας*) which appeared in Byzantium in 1252, a book of arithmetic called *The [so-called] Great Calculation according to the Indians* (*Ψηφιφορία κατ' Ινδούς, η λεγόμενη μεγάλη*), which treated the four fundamental mathematical operations, the extraction of the square root, and employed the zero to enable representation of very large numbers. The so-called "Indo-Arabic" numerals appeared in Byzantium for the first time in the 12th century in a commentary to Euclid. They were introduced together with the Arab methods for performing the basic operations by Leonardo of Pisa (known as Fibonacci) in his work *Liber abaci*, when he visited Constantinople in the late 12th century to discuss arithmetical and algebraic issues with Byzantine scholars.[22]

Planudes was also interested in astronomical and geographic works by the Greeks. He managed to improve the *Phaenomena* of Aratus thanks to his excellent knowledge of the Almagest, and he collected the old manuscripts of Strabo. He searched out manuscripts of Ptolemy's *Geography*, and his quest was rewarded in 1295. As he explained in one of his letters, when he finally found the manuscripts he knew that they were the lost work he was looking for, but was disappointed when he saw that there were no maps. Thus by relying on the text of Ptolemy, he was obliged to draw a world map himself, to which only the commentary has survived. The Emperor Andronicus II Palaeologus, impressed by Planudes' map, asked the Patriarch of Alexandria Athanasius for an edited copy of the *Geography* of Ptolemy with its maps. This was probably the famed copy in the Vatican, whose maps are considered world masterpieces of cartography.[23]

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21 This was the Vienna philosophical Greek Codex 65 that included exercises and problems similar to those of the Parisinus supplementary Greek codex 387. For a transcription, commentary and philological editing of this manuscript see Maria Chalkou, 2006.

22 Two geometry books found in the 10th century were attributed to the "last Roman scholar" Boethius. The first of these contained what was practically a word-for-word translation of the terms in Euclid's *Elements*. At the end of the book is a page (if it is authentic) where the author hints that the invention and use of the symbols 1, 2, 3, …, 9 goes back to Greece and the Neopythagoreans, whence they became known to the Persians, Indians, and then to the Byzantines and Arabs, via whom they arrived in Spain.

23 For further background on Byzantine Cartography, see Dilke (1987) and Livieratos (1998).

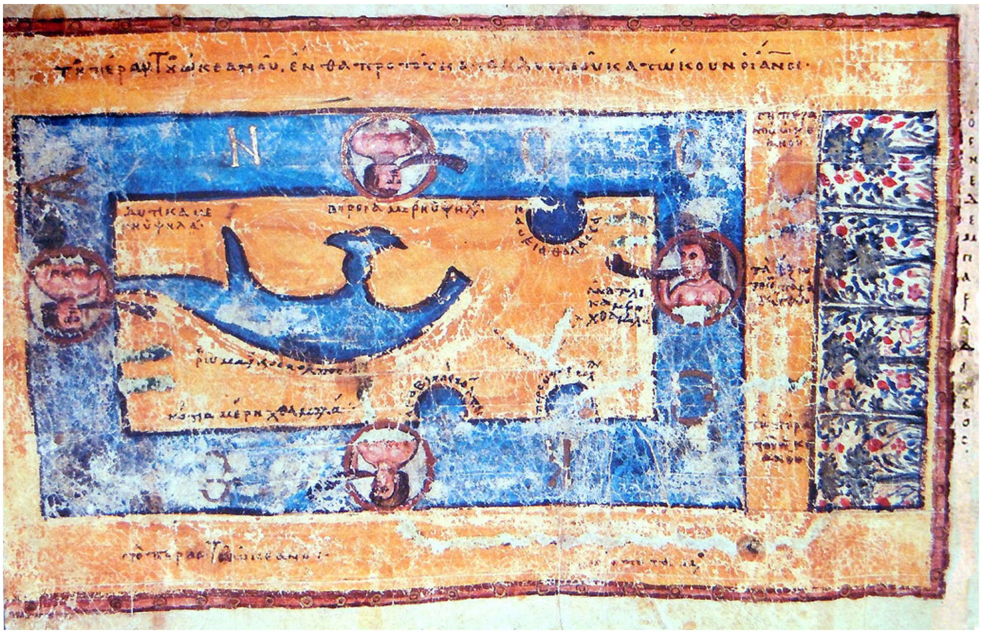

***Figure 5**. World map, by Cosmas Indicopleustes. The rectangular inhabited world is surrounded by an ocean. To the east, beyond the ocean, is paradise. Beyond the ocean at the top (south) is the uninhabited world. (11th century, Codex Sinaiticus graecus 1186, St. Katherine's monastery, Sinai)*

The maps of Ptolemy were redrawn until the waning years of the empire. The *Geography* (*Γεωγραφική Υφήγησις*) was in the library of Manuel Bryennius. Apart from Agathemerus, who wrote the very average *Brief Sketch of Geography* (*Υποτύπωσιν γεωγραφίας*), Nicephorus Blemmydes and Nicephorus Gregoras are also worth mentioning. Blemmydes, relying on the work of Dionysius Periegetes of the 2nd or 3rd century BC, wrote a *Synoptic Geography* (*Γεωγραφία Συνοπτική*) and *Another history of the world in brief for an Orthodox king* (*Ετέρα ιστορία περί της γης εν συνόψει προς τινά βασιλέα ορθόδοξον*), which dealt with the size and the curvature of Earth and the seven *climata*.

Nicephorus Gregoras, a student of Theodore Metochites, 14th-century Byzantium's preeminent intellectual and political figure, wrote two books on the astrolabe, which he himself used for observations. The first of these was dedicated to the construction of the astrolabe, and the second to its theoretical underpinnings. Nicephorus Gregoras was a mapmaker, and provided a commentary on Ptolemy's *Geography*. Isaac Argyros, a student of Gregoras, wrote a handbook of Geodesy similar to the pseudo-Heronian texts as well as a text on constructing an astrolabe, based on the work by his teacher.

Fifteen Byzantine treatises on the astrolabe have survived, including those by

Figure 6. *The oldest surviving Ptolemaic world map created in Constantinople at the beginning of the 14th century. It decorated the Ptolemaic Vatopedion Codex 655. The geographical coordinates of the meridians and parallels are marked on the margins and reviews referred to climata appears at the left edge. The heads of the ten winds are depicted of perimeter. It was stolen by Constantine Symeonidis with other sheets of the manuscript and was sold to the British Museum in 1853.*

Theodore Metochites, Ioannes Kamateros, Nicephorus Gregoras, Isaac Argyros, and Barlaam in addition to others by anonymous authors. Of these, thirteen were written in the Palaelogan period. Unfortunately, despite this rich tradition, only one astrolabe has survived. Found in Brescia, it was built in 1062 for Sergius the Persian, who held the Byzantine titles of *protospatharius* and consul. This instrument was adopted by the Arabs, who came to know it during the seventh century in the city of Harran in Syria, which was the most important center for the dissemination of Hellenistic knowledge. It was used in astronomy and astrology, to determine the position of the planets in the heavens in combination with specific tables to find the times for the five daily prayers, in geodesy - e.g., to determine latitude, to find the direction to Mecca, and in simple topographical problems to measure angles in place of a *dioptra*.

During Byzantine times there was tremendous interest in all geographic knowledge which was of practical significance or which served ecclesiastical, political, and commercial purposes: maps, itineraries, and guides. Itineraries and travelogues presupposed the existence of maps, and were published for the use of citizens primarily for religious reasons, e.g. Byzantine itineraries of the Holy Land were published for the faithful traveling to Jerusalem.

Periploes, which like the practical itineraries which had recorded distances between cities, ports, islands, etc. in Greece since antiquity, also continued to be published in Byzantine times, and corresponded to today's manuals for ships' pilots and nautical charts. The Greek geographer Marcian of Heraclea (Marcianus Heracleensis), author of a work called *Periplous of foreign seas* (*Περίπλους της έξω θαλάσσης*; *Periplus maris exteri*), who as he himself mentions employed Ptolemy as his source, lived in the early 4th century. The *Periplous of the Euxine Sea* (*Περίπλους Πόντου Ευξείνου*), which has been erroneously attributed to Arrian, was written after the second half of the 6th century, as was what was essentially a copy of it by an anonymous writer entitled *A brief reckoning of the entire inhabited world* (*Αναμέτρησις της οικουμένης πάσης κατά σύνοψιν*); this work, however, is difficult to date. In the introduction, the circumference of the earth is defined as 252,000 stades, and the length-width of the inhabited earth as 83,000 and 3500 stades. Another work, *Stadiodromicon, or the measurement of distances from Byzantium to Crete* (*Σταδιοδρομικόν, ήτοι αναμέτρησις αποστάσεων από Βυζαντίου εις Κρήτην*), was included in the Constantine Porphyrogenitus' *On the Governance of the Empire* (*Περί της βασιλείου τάξεως*; *De administrando imperio*), which was written in 949 during preparations for the campaign against the Saracens in Crete.

The most important of the Byzantine books of this type, whose author is unknown, is preserved in a manuscript in Madrid under the title *Stadiasmos, or Periplus of the great sea* (*Σταδιασμός, ήτοι περίπλους της μεγάλης θαλάσσης*). This handbook, which is actually a copy of an earlier Greek work, describes the *periplous* of the Mediterranean coast, enriched with information concerning distances between ports, descriptions of these, the locations of reefs and shoals, places to obtain supplies, etc. The points of reference for calculating distances are Rhodes, Delos, and other islands in the Aegean Archipelago.

5. Instead of an epilogue

In concluding our account of the work of Byzantine teachers and eponymous geometers, we should note - as many historians of science have claimed - that while perhaps they added nothing of extraordinary importance to the work of the Greek mathematicians, nonetheless their contribution to the preservation of all that had been achieved in antiquity, both in Classical Greece and elsewhere (above all, Hellenistic Alexandria) is unquestionable. Eutocius, John Philoponus, Simplicius, Leo the Wise, Michael Psellos, Isaac Argyros, John (Ioannes) Pediasimos, Maximus Planudes, and others established in Byzantium, preserved and disseminated the major work of Classical Greek and Hellenistic antiquity to the West, either directly through Greeks who traveled there, or through the Arabs. This process of restoring Greek technology and science reached its apogee during the Renaissance.

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