Coordinate Kinematic models in the International Terrestrial Reference Frame releases

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Abstract: In the latest International Terrestrial Reference System realization (ITRF2014) combination model, new types of displacements have been introduced by means of mathematical functions. The addition of these functions has led to the implementation of new constraints to define the reference frame. This work was anticipated by A. Dermanis (2008) who derived constraint equations for different kinematic models.

This paper presents the fundamental theoretical concepts that have been used to derive the latest International Terrestrial Reference Frame (ITRF). A new physical interpretation of the partial inner constraints involving transformation parameters is presented to supplement earlier work. By reviewing the various possibilities that could have been implemented to enhance the ITRF coordinate variations, this paper justifies the ITRF2014 chosen kinematic model and why it still does not include functions suggested by Dermanis (2008).

Introduction

Determining accurate coordinates at the Earth's surface is not straightforward since the reference system axes are not accessible in practice. As a consequence, a set of physical points whose coordinates are known is commonly used to materialize the terrestrial reference system and gives an implicit access to the system axes. This set of coordinates supplemented by statistical indicators is called Terrestrial Reference Frame. Those coordinates include a part of conventional information for defining the position of the origin of the frame, the orientation of its axes with respect to the crust and its scale (unit of length). One wishes that this frame be in co-rotation with the Earth for positioning objects at the Earth's surface.

As the Earth is not rigid, points at the Earth's surface exhibit relative motions which indubitably lead to coordinate variations in the Terrestrial Reference Frame whatever its definition is. Nowadays, relative positions between points are determined by space geodesy which provides the best precision for baselines at intercontinental distances. And all space geodetic measurements involve objects whose kinematic and dynamic descriptions are simplified in a celestial reference frame. Thus, the rotation of the Earth and coordinates of the tracking stations are solved

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for simultaneously in space geodetic software packages. This simultaneous estimation leads to a rank deficiency of the equation system to be solved since any change of the Earth's rotation vector can be canceled by an opposite rotation of the coordinate sets. Thus, constraints are added to define the reference frame of the station coordinates which make the estimated rotation vector of the Earth's motion in space fully dependent on the frame definition.

Dermanis (2000) revisited this issue of finding the most suitable reference frame in an elegant way. Assuming that the coordinates of station i, written $\mathbf{x}_i(t)$, are known in an arbitrary frame, it is possible to define coordinates in a new frame $\mathbf{x}_i(t)$ using a similarity transformation $\mathbf{x}_i'(t) = s(t) \mathbf{R}(t) \mathbf{x}_i(t) + \mathbf{T}(t)$. The fundamental problem of reference system consists in finding at each epoch t the scale factor s(t), the rotation matrix $\mathbf{R}(t)$ and the translation $\mathbf{T}(t)$ to be applied according to an optimal criterion. This criterion has been historically chosen in order to simplify the equation of the Earth's rotation in space (Munk and MacDonald, 1960; Gross, 2015) without introducing large coordinate changes in the terrestrial frames (Dermanis, 2001). Tisserand frames are ideal terrestrial frames since they fulfill this condition (Kovalevsky and Mueller, 1984). They have an origin at the Earth's center of mass and an orientation which is such that the contribution of the whole Earth's deformations to the angular momentum –the relative angular momentum– is equal to zero. Among all existing frames, these frames minimize the kinematic energy and thus involve less coordinate variations (Boucher, 1989).

In order to apply these concepts, the distribution of masses as well as the associated deformations need to be known at any point of the Earth's surface and of the Earth's interior. Whereas space geodesy is able to determine the Earth's center of mass position without this knowledge, this knowledge is required to define the Tisserand frame orientation time evolution. Also because geodetic stations are located at the Earth's surface, a modification of the Tisserand conditions has been adopted by the International Earth Rotation and Reference Systems Service. The International Terrestrial Reference System (ITRS) is specified as follows (Petit and Luzum, 2010, Chapter 4):

- 1. It is geocentric, its origin being the center of mass for the whole Earth, including oceans and atmosphere;
- 2. The unit of length is the meter (SI). The scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modeling;
- 3. Its orientation was initially given by the BIH orientation at 1984.0;
- 4. The time evolution of the orientation is ensured by using a no-net-rotation condition with regards to horizontal tectonic motions over the whole Earth.

The last point is equivalent to nullify the relative angular momentum of the Earth's crust only. As discussed by Altamimi and Dermanis (2012b), this can be addressed by a kinematic approach or by an algebraic approach. The algebraic approach is the one that has been used to define the orientation of the ITRF. It relies on an a priori knowledge of the Earth's surface motion through a geophysical model and on the use of inner constraints for ITRF2000 (Altamimi et al., 2001). It also requires that the kinematic model for coordinate variations, namely piecewise linear functions, is known a priori. Conversely, Athanasios Dermanis has extensively developed the concept of the kinematic approach (Dermanis, 2000; Dermanis, 2001; Dermanis, 2003). This consists in the discrete computation of the relative angular momentum integral over the station network. In particular, he has developed the datum constraint equations by anticipating future changes in the ITRF kinematic model, considering that coordinate variations should not be linear anymore (Dermanis, 2008). Thus, polynomial functions, Fourier series and splines were suggested.

We propose in this paper to review the adopted model of ITRF2014 for coordinate variations and explain why some of Dermanis (2008) suggestions are not yet implemented. In section I, ITRF previous releases are shortly described and the ITRF model is presented. Altamimi and Dermanis (2012a; 2012b) theoretical work on datum constraint is recalled and a new interpretation of partial inner constraints involving transformation parameters is provided. In section II, the ITRF2014 model is discussed and finally new seasonal parameters are discussed in section III.

I ITRF as a parametric frame

I.1 Regularized frame

The instantaneous coordinates of a point in a terrestrial reference frame can be described as follows according to the IERS conventions (Petit and Luzum, 2010, Chapter 4):

$$
\mathbf{x}(t) = \mathbf{x}_r(t) + \Sigma \Delta \mathbf{x}(t) \tag{1}
$$

where $\mathbf{x}_r(t)$ are the coordinates at epoch t in the terrestrial reference frame, called regularized coordinates, and $ΣΔx(t)$ is the sum of conventional corrections.

Up to now, the conventional corrections include displacements at rather short periods from solid Earth tides, ocean tide loading, S1 and S2 atmospheric loading and pole tides (Petit and Luzum, 2010; IERS, 2015). Any other phenomenon that causes long-term crust motion will then be reported in the reference coordinates $\mathbf{x}_r(t)$. Thus they reflect tectonic motions, post-glacial rebound or other effects.

ITRF coordinates are regularized coordinates described by simple mathematical functions as described in the next section. They are the result of an adjustment of space geodetic data including Global Navigation Satellite Systems (GNSS), Very long Baseline Interferometry (VLBI), Satellite and Lunar Laser Ranging (SLR and LLR) and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) (Boucher et al., 1999; Altamimi et al. 2002, 2007, 2011, 2016). Thus, a priori conventional models $\Sigma \Delta x(t)$ have been included in the processing of the data which means that reference coordinates implicitly include conventions with respect to some adopted models. A relevant example is the handling of permanent deformations related to solid Earth tides (Petit and Luzum, 2010, chapter 1). A similar situation occurs with technique-specific corrections such as antenna phase center corrections for GNSS (Schmid et al., 2016). Any change in the used corrections impacts station coordinates, that's why the International GNSS Service (IGS) publishes updates of ITRF such as IGS08 for ITRF2008 every time a new correction file is released (Rebischung et al., 2012).

From a user point of view, it means that terrestrial reference frame coordinates are related to certain conventions so that users should take care of the conventional corrections they add to them or use with them. For this reason, there is no attempt to correct technique-specific coordinate offsets in the published coordinates $\mathbf{x}_r(t)$ if the conventional models are not changed.

II.2 History of ITRF products

There have been 13 ITRS realizations published since 1988 (Petit and Luzum, 2010, Chapter 4). Figure 1 provides a summary of the input data used in terms of space geodetic techniques and data span for the most recent published frames. As can be noticed, the input data span increases for each realization making the most recent ITRF superior to its predecessors.

Fig. 1. Time spans of recent ITRF input data

The kinematic model of coordinate variations $\mathbf{x}_r(t)$ has evolved with time starting with constant coordinates in the earlier period. Although a geophysical velocity model was advised to be used to predict coordinates in the past and future, this method implied regular updates of the terrestrial frame since the geophysical model used differs from observations. From ITRF91 (Boucher et al., 1992), velocities were estimated and provided to the users. The following kinematic model for reference coordinates was adopted:

$$
\mathbf{x}_{r}(t) = \mathbf{x}_{c0} + \Sigma \delta \mathbf{x}_{i} \mathbf{H}(t - t_{x,i}) + (t - t_{0}) \mathbf{v}_{c0} + \Sigma \delta \mathbf{v}_{i} \mathbf{H}(t - t_{v,i})
$$
(2)

where $H(t)$ is the Heaviside function (equal to 0 when $t < 0$; equal to 1 when $t \ge 0$), t_0 is the reference epoch of the coordinates and $t_{x,i}$ $t_{v,i}$ are the epochs of discontinuities. In ITRF tables, coordinates are supplied to users as $(x_c(t), y_c(t))$ such as:

$$
\mathbf{x}_{c}(t) = \mathbf{x}_{c0} + \Sigma \delta \mathbf{x}_{i} \ H(t - tx_{x,i})
$$

\n
$$
\mathbf{v}_{c}(t) = \mathbf{v}_{c0} + \Sigma \delta \mathbf{v}_{i} \ H(t - tx_{x,i})
$$

\n
$$
\mathbf{x}_{r}(t) = \mathbf{x}_{c}(t) + (t - t_{0}) \ \mathbf{v}_{c}(t)
$$
\n(3)

The dependency of the ITRF coordinates on the epoch is handled by means of the solution number, the so-called "soln". The soln is an integer which is associated to a time interval, so that any linear segment of the piece-wise linear function is identified by a unique integer.

Although computation methods have evolved, including the use of variance/covariance information since ITRF94 (Boucher et al., 1996), the use of time series of frames as input data since ITRF2005 (Altamimi et al., 2007), there has been no change in the description of station coordinate time evolutions up to ITRF2014.

II.3 Estimation model

II.3.1 Generalities

The ITRF estimation model for recent releases has been published in Altamimi et al. (2002; 2007) and more deeply conceptualized in Altamimi and Dermanis (2012b). In this section, we complete Altamimi and Dermanis (2012b) discussion by a complementary physical interpretation of the partial inner constraints also called internal constraints.

Since ITRF2005, time series of coordinates for a set of stations were provided for each measurement technique. Whereas all data could be in theory combined in one step (Equation 1 of Altamimi et al., 2007) to provide coordinates and velocities for each station in a well-defined reference frame, a two-step approach was adopted for better data editing and to facilitate the computation.

In the first step, positions, velocities and discontinuities in positions and velocities, hereafter called coordinates, are computed for every station for each technique independently. This first step is called stacking.

The second step, called combination, makes a proper average of all available data to provide coordinates in the same reference frame for all techniques making use of:

- coordinates of stations with their associated variance/covariance matrices from each technique as the results of the stacking step,
- the relative positions of instruments from different techniques located in the same observatory –the so– called local tie vectors determined by topometry, levelling and GPS technique,
- •pseudo-observations to define the reference frame of the output coordinates.

II.3.2 Equation of state in g

The observation equation of the stacking is given below for station i in the input station coordinate set s, following Altamimi and Dermanis (2012a) notations:

$$
\mathbf{x}_{s, i}(t) = \mathbf{x}_{c, i} + (t - t_0) \mathbf{v}_{c, i} +\n+ \mathbf{t}_k + [\mathbf{x}_i^{ap} \times] \mathbf{\theta}_k + s_k \mathbf{x}_i^{ap}
$$
\n(4)

where t_k , s_k , θ_k are the transformation parameters, respectively a translation vector, a scale factor offset (with respect to 1) and a rotation vector – vectors are written in bold. Transformation parameters are assumed to be small so that the similarity mentioned in the introduction has been linearized around a priori position x_i ^{ap} for station *i*. Thus, the rotation matrix becomes $I + [x^{ap} x]$ where I is the identity matrix. t_0 is the reference epoch of the estimated coordinates $x_{c,i}$ and $v_{c,i}$. In equation 4, the dependency of $\mathbf{x}_{c,i}$ and $\mathbf{v}_{c,i}$ on the epoch has been omitted but the appropriate coordinate parameters should be chosen in case of position and velocity discontinuities. The simultaneous estimation of transformation parameters together with station positions and velocities makes the system rank deficient (Altamimi et al., 2000) where the rank deficiency corresponds to the number of parameters that are necessary for the definition of the output reference frame. Constraints can be either added to parameters $\mathbf{x}_{c,i}$ and $\mathbf{v}_{c,i}$ –minimum constraints also called partial inner constraints involving coordinates– or to parameters t_k , s_k , θ_k –internal constraints also called partial inner constraints involving transformation parameters.

In the current ITRF construction, both types of constraints are used, depending on the datum parameter to be defined. For example, internal constraints are used for translation and scale parameters and minimum constraints for orientation parameters (see Altamimi et al. 2007; 2011; 2016).

II.3.3 Partial inner constraints involving coordinates

Minimum constraints as defined by Altamimi et al. (2001) are algebraic constraints that are imposed on coordinates to force them to have zero transformation parameters with respect to a reference coordinate dataset.

They can be derived from inner constraints that involve the a priori coordinates as reference coordinates of the constraints. The latter read as follows (Dermanis, 2000):

$$
\mathbf{E}^{\mathrm{T}}(\mathbf{x}_{c} - \mathbf{x}^{\mathrm{ap}}) = 0
$$

$$
\mathbf{E}^{\mathrm{T}}(\mathbf{v}_{c} - \mathbf{v}^{\mathrm{ap}}) = 0
$$
 (5)

Where E is the matrix of partial derivatives of coordinates with respect to transformation parameters t_k , s_k , θ_k . \mathbf{x}^{ap} and \mathbf{v}^{ap} are the vectors of a priori coordinates. In the scope of Altamimi and Dermanis (2012b) developments, this constraint is a partial inner constraint involving only coordinates since the problem to be solved here also involves transformation parameters. However, given a constraint equation, any new constraint equation can be derived by multiplying it by an invertible matrix (Vanícek and Krakiwsky, 1986). Minimum constraints as defined by Altamimi et al. (2001) are given as follows:

$$
(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\mathbf{x}_{c} - \mathbf{x}^{ap}) = (\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\mathbf{x}_{ex} - \mathbf{x}^{ap})
$$
(6)

$$
(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\mathbf{v}_{c} - \mathbf{v}^{ap}) = (\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\mathbf{v}_{ex} - \mathbf{v}^{ap})
$$

where (x_{ex}, y_{ex}) is an external coordinate set used as reference for the constraints.

II.3.4 Partial inner constraints involving transformation parameters

An alternative type of constraints involves the transformation parameters, in the case of stacking of station position time series. Since any input coordinate dataset at epoch k requires transformation parameters (t_k , s_k , θ_k) at the same epoch, time series of transformation parameters are actually estimated. If one wants to obtain a time series of transformation parameters with zero mean and zero drift, the following equation must hold for a parameter time series **p**:

$$
(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{p} = 0 \quad ; \quad \mathbf{A} = \begin{pmatrix} 1 & \mathbf{t}_1 - \mathbf{t}_0 \\ \vdots & \vdots \\ 1 & \mathbf{t}_K - \mathbf{t}_0 \end{pmatrix} \quad ; \quad \mathbf{p} = \begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_K \end{pmatrix} \tag{7}
$$

This equation can be obtained by nullifying the least squares estimates of the constant and drift of a linear regression. By performing the computation, the first equation can be simplified as follows:

$$
\sum_{k} p_k = 0
$$
\n
$$
\sum_{k} (t_k - t_0) p_k = 0
$$
\n(8)

This equation is the partial inner constraint involving only transformation parameters (Altamimi et al., 2007, Dermanis 2012b).

For a more physical interpretation of this type of inner constraints, we suggest working on a simplification of equation (4) where only translations would be involved:

$$
\mathbf{x}_{s,i}(t_k) = \mathbf{x}_{c,i} + (t_k - t_0)\mathbf{v}_{c,i} + \mathbf{t}_k
$$
 (9)

The problem of solution numbers is easily handled since a new linear segment can be handled similarly by considering it as a new station. We provide an analytical solution of equation (9) under conditional least squares, considering K sets of coordinates as observations and additionally assuming that

- t_0 is the average time over all t_k ,
- the condition equation (8) applies to the three components of the time series of vectors t_k ,
- the input sets of coordinates have identity variance-covariance matrices.

It can be shown that the stacked coordinates are in this case the following (Collilieux, 2008):

$$
\hat{\mathbf{x}}_{c,i} = \frac{1}{K} \sum_{k} \mathbf{x}_{s,i}(t_k)
$$
\n
$$
\hat{\mathbf{v}}_{c,i} = \frac{\sum_{k} (t_k - t_0) \mathbf{x}_{s,i}(t_k)}{\sum_{k} (t_k - t_0)^2}
$$
\n(10)

Thus, the estimated coordinates are those obtained from a standard linear regression without any translation parameters. The origin of the obtained frame has no drift and no mean offset with respect to the input frames due to equation 8. This result reinforces that adding constraints as in equation (8) allows defining a mean frame. Moreover, as proposed by Altamimi et al., (2011; 2016), it is possible to apply the constraint equation 8 on a subset of time indices t_k corresponding to more reliable input coordinate sets, which allows to introduce all the other coordinate sets without affecting the frame definition of the combined coordinate set. In practice, those types of constraints are chosen to define the frame of the technique stacked solutions that contribute to the ITRF frame definition.

II.3.5 Combination

The second-step of the ITRF processing is the combination. The stacked coordinates from the different techniques are combined with local ties. In this process, transformation parameters between each coordinate dataset and the coordinates to be computed are estimated (Equation 1 of Altamimi et al., 2007). 14 constraint equations have to be added to define the frame of the output coordinates since positions at the reference epoch and velocities are linearly related to transformation parameters in the combination model. Because certain space geodetic techniques are sensitive to the origin and scale, defining the origin and scale of the combined coordinates consists in constraining some estimated transformation parameters to zero (Altamimi et al., 2001).

Algebraic constraints (see II.3.3) are added to $\mathbf{x}_{c}(t)$ to define the orientation of the combined frame at the reference epoch and to $v_c(t)$ to define the orientation time evolution. Since ITRF2005, the conventional time evolution of the frame orientation has been defined through inner constraints with respect to the previously released velocity field. For ITRF2000, the geophysical model NNR-NUVEL1A, that verifies the no-net-rotation condition as specified in the ITRS system definition, has been used as reference velocity field for the orientation time evolution constraint. The choice of the orientation time evolution impacts studies that interpret polar motion drift. For users interested in that particular aspect, it is still possible to apply afterward a rotation contribution as done in Argus and Gross (2004) in order to change the orientation of the ITRF, or to use the kinematic methods suggested by Dermanis (2001; 2008).

II.4 Deficiency of the kinematic model in previous ITRF

There have been intensive researches on the analysis of position time series of space geodetic stations. For example, for GNSS, studies by Dong et al., (2002), Williams et al. (2004) and Ray et al., (2008, 2013) are worth mentioning. In these studies, regularized positions and velocities are usually computed and removed from the determined coordinates to study their non-linear variations.

The following conclusions are worth reporting:

- Seasonal signals are evidenced in all technique estimated coordinates (Collilieux et al., 2007; Altamimi and Collilieux, 2010).
- A significant part of seasonal signals is related to real crust motions, namely loading effects that are the elastic deformations of the Earth due to mass transports in its fluid layers (e.g. Mangiarotti et al., 2001, van dam et al., 2001; Dong et al., 2002; Petrov and Boy, 2004; Collilieux et al., 2010; Fritsche et al., 2011), see section III.1.
- Loading effects, especially due to ice melting, can cause large non-stationary signals as found by Khan et al. (2008) in Greenland. See figure 2b) for an up-

dated plot, where a seasonal signal is also visible.

- • Non-tidal loading corrections are currently not recommended by the IERS (Petit and Luzum, 2010, Chapter 7).
- • Technique specific errors are likely to remain in space geodetic technique coordinates (Altamimi and Collilieux, 2010; Sarti et al., 2011; Ray et al., 2013; Appleby et al., 2016)
- Co-seismic signals affect a large number of stations (Tregoning et al. 2013, Métivier et al., 2014).
- •Silent Earthquakes affect at least one ITRF site (Schwartz and Rokosky, 2007)
- • Post-seismic signals affect a significant number of stations, especially since the three last giant earthquakes (Altamimi et al., 2016).

Fig. 2 a) GNSS position time series at THU3 (Greenland).
b) GNSS position time series at SAMP (Indonesia) b) GNSS position time series at SAMP (Indonesia). Source: http://itrf.ign.fr

III Station motion modeling in ITRF2014

III.1 Non-tidal loading effects

III.1.1 Background on non-tidal loading effects

The elastic deformation of the Earth in response to a surface load can be computed using either Green's functions or a spherical harmonic expansion (Farrell, 1972). In order to predict deformations at a point of coordinates $\Omega = (\lambda, \varphi)$, both the elastic properties of the Earth (described by Load Love numbers for a spherically symmetric, non-rotating, perfectly elastic and isotropic (Dahlen and Tromp, 1998)) and the distribution of the load, i.e. its surface density $\sigma(\Omega)$, have to be known. If $\sigma(\Omega)$ is provided as a spherical harmonic expansion, it can be written as follows:

$$
\sigma(\Omega) = \sum_{m=0}^{\infty} \sigma_{l,m}^{c} R_{n,m}(\Omega) + \sigma_{l,m}^{s} S_{n,m}(\Omega)
$$
\n(11)

where $R_{n,m}(\Omega)$ and $S_{n,m}(\Omega)$ are not normalized and related to associated Legendre polynomials by $R_{n,m}(\Omega) = P_{nm}(\sin \varphi) \cos(m\lambda)$ and $S_{n,m}(\Omega) = P_{nm}(\sin \varphi) \sin(m\lambda)$. The horizontal and vertical displacements can be computed by (Blewitt et al., 2001; Blewitt, 2003):

$$
\vec{v}_l(\Omega) = \frac{4\pi R^3}{M} \sum_{n=0}^{\infty} \frac{l'_n}{2n+1} \sum_{m=0}^n \sigma_{l,m}^c \vec{\nabla} R_{n,m}(\Omega) + \sigma_{l,m}^s \vec{\nabla} S_{n,m}(\Omega)
$$
(12)

$$
u_{l}(\Omega) = \frac{4\pi R^{3}}{M} \sum_{n=0}^{\infty} \frac{h'_{n}}{2n+1} \sum_{m=0}^{n} \sigma_{l,m}^{c} R_{n,m}(\Omega) + \sigma_{l,m}^{s} S_{n,m}(\Omega)
$$
 (13)

with l_n and h'_n the load Love numbers of degree n, R the mean radius, M the mass of the Earth and $\vec{\nabla}$ is the unit surface gradient operator (Blewitt and Clarke, 2003).

Thus, forward non-tidal loading models can be derived using numerical models of the mass distributions. Most of the time, forward models are generated by separating fluid layers (van Dam, 1994; van dam et al., 2001; van Dam et al., 2012) such as the atmosphere, ocean non-tidal mass transport, continental hydrology and ice sheet mass variations. However, care should be taken when summing models from various sources (Clarke et al., 2005) since interactions exist between the different fluid layers (exchange of water). The different models are available with various time samplings, from 3 hours to 1 month, and various spatial resolutions.

As discussed by Blewitt (2003), load Love numbers of degree-1 are dependent on the reference frame of the computed displacement field which are most often the center of mass of the solid Earth (CE), the geometric center of the solid Earth's surface (CF) or the center of mass of the whole Earth system (CM). CM is the natural frame for satellite techniques. The variations of the vector CM with respect to CF are usually called geocenter motion and the contribution of the gravitational attraction of the mass variations at the Earth's surface and associated elastic deformations can also be computed by using only degree-1 coefficients of $\sigma(\Omega)$:

$$
[\vec{\tau}_{CF}]_{CM} = \left(\frac{1}{3} [\mathbf{h}'_1 + 2l'_1]_{CE} - 1\right) \frac{4\pi \mathbf{R}^3}{3M} \begin{pmatrix} \sigma_{11}^c \\ \sigma_{11}^s \\ \sigma_{10}^c \end{pmatrix}
$$
(14)

The variations of CF with respect to CE are very small, about 2% of the geocenter motion magnitude for elastic deformations (Dong et al., 1997). As stations are attached to the Earth's surface and because the analysis of translation parameters partial derivatives makes appear the barycenter of the station network (Dong et al., 2003; Collilieux et al., 2009), net translations of a geodetic network with respect to mean coordinates are close to the opposite of geocenter motion. It is fundamental to understand that the geocenter motion is a part of the loading effects themselves and should be ideally treated simultaneously. Recent annual geocenter motion estimates predict translations with amplitudes at the level of 3-4 mm along the X, Y and Z components (Collilieux et al., 2009). Thus, mean coordinate variations in the CM frames are close to the geocenter motion magnitude (Dong et al., 2003).

III.1.2 Handling of non-tidal loading effects

In the ITRF releases, non-tidal loading effects have been ignored since they were expected to be averaged out when computing positions and velocities from long enough datasets (Blewitt and Lavallée, 2002). However, for stations with a few observations, station positions and velocities have been reported to be biased by loading deformations (Collilieux et al., 2010), affecting station coordinates from different techniques at co-location sites. This also means that if one wants to sum ITRF coordinates with a non-tidal loading displacement model, small biases may be generated since ITRF coordinates may have already captured a possible drift related to non-tidal loading effects.

There are mainly two ways to handle non-tidal loading effects according to equation (1) :

- to correct for a non-tidal loading model, i.e. to model the non-tidal loading effect in the term $\Sigma \Delta x(t)$,
- to add new parameters in the regularized coordinate kinematic model $\mathbf{x}_r(t)$.

The first alternative is in theory the most rigorous. Indeed, it allows accounting for the effects over the whole power spectrum from sub-daily periods to long periods. When processing data for determining a reference frame, loading corrections should be ideally made at the observation level, which in addition allows modeling the induced effect on the geopotential for satellite techniques. The inclusion of such models has been shown to reduce observation residuals although non-tidal loading models are not free of errors, for example evaluated at the level of 15% of the effect for non-tidal atmospheric loading (Petrov and Boy, 2004). Hydrology models, for instance, show large discrepancies, particularly in the treatment of glaciers, ice sheets and lakes (e.g. Rodell et al., 2004). Due to the data processing time that can be important especially for GNSS and because they are quicker and more flexible, a posteriori corrections which consist in correcting coordinates by the mean load effect over the whole data integration time have been also studied (Tregoning and Watson, 2009). However, Dach et al. (2011) have suggested a relevant way to add geometric loading model corrections in the software packages while providing a rigorous way to remove them. They have added parameters to scale the non-tidal loading model corrections in the normal equations which allow applying the loading model or not by fixing them to zero or one when inverting the normal system.

The second alternative consists in estimating the loading displacement from station coordinates based on equation (12) and (13) as proposed by Blewitt et al. (2001). Rülke et al. (2008) were the first to estimate station positions and velocities simultaneously with load spherical harmonic coefficients. They estimated them on a monthly basis up to degree 6. The method allows modeling the geocenter motion effect with its associated degree-1 deformation with a relevant time sampling but the truncation degree used was too small to capture all non-tidal loading signals. Increasing the truncation degree would require to use additional datasets such as GRACE data (Wu et al., 2006), but the time span of the mission starting in 2002 does not cover the whole ITRF data period (see figure 1).

Dermanis (2008) suggestion of extending the station coordinate kinematic model to other function types is a relevant alternative since the main observed coordinate variations can be captured by parametric functions. Polynomials, splines and harmonic functions were suggested. Polynomial functions could for example capture the accelerations observed in figure 2 for station THU3 in Greenland. However, it is not certain that coordinates will follow the same pattern after the publication of the regularized coordinates. Indeed, the kinematic coordinate model is used to extrapolate coordinates in the future for routine precise orbit determination. Spline functions also fail to provide reliable extrapolations of coordinates although they provide an excellent fit to the data. Periodic functions are more relevant and were already used in reference frame analyses (Meisel et al., 2009). They have the advantage of providing a more reliable estimation of station velocities (Blewitt and Lavallée, 2001). In addition, as many position discontinuities are evidenced in particular in GNSS position time series, the addition of periodic parameters in the functional model improves the estimation of offsets. The main drawback is that periodic terms may also absorb technique-specific errors. In addition, they are not constant in time (Chen et al., 2013), but their amplitudes vary moderately. Some cases of drastic seasonal signal changes in GPS coordinate series have been reported (Ray, 2006) but they were related to equipment changes.

III.2 ITRF2014 coordinate modeling and estimation model

For ITRF2014, the ITRS product center explicitly asked not to correct for non-tidal models at the observation level. But tests have been carried out to correct for nontidal atmospheric loading a posteriori (Altamimi et al., 2016) during the ITRF preparation. However, the adoption of periodic parameters was preferred since they impact most significantly the estimated velocities. That's also why only annual and semi-annual periods have been considered, since other frequencies have a smaller impact on the estimated reference positions and velocities (Altamimi et al., 2016). However, seasonal parameters were not published as part of the coordinate kinematic model, see discussion in section III.3.

New functions have been used to model post-seismic deformations at stations where it was needed. The reader is referred to the appendix C of Altamimi et al. (2016) for a complete description of the post-seismic models that were adopted. A model, hereafter written δPSD(t), is made of a combination of exponential and logarithmic functions with various relaxation times. Not more than two functions are used per earthquake and coordinate component in a frame directed along the parallel, the meridian and the normal at the ellipsoid at the station location. For stations far from the earthquake epicenter, δPSD(t) is set to zero since the postseismic motion can be represented by position offsets and constant velocity changes, as done in previous ITRF releases. Figure 2b) shows a station affected by significant post-seismic signal. The piece-wise linear function part from the kinematic model is shown in green and the complete model including the correction δPSD(t) in red. As evidenced in this figure, users need to take the post-seismic deformation models δPSD(t) into account when using ITRF2014 positions and velocities.

Equation (4) of the stacking has then been modified as follows:

$$
\mathbf{x}_{s,i}(t) - \delta \text{PSD}(t) = \mathbf{x}_{c,i} + (t - t_0) \mathbf{v}_{c,i} \n+ \sum_{j} \mathbf{a}_i^j \cos \omega_j (t - t_0) + \mathbf{b}_i^j \sin \omega_j (t - t_0) \n+ \mathbf{t}_k + [\mathbf{x}_i^{\text{ap}} \times] \mathbf{\theta}_k + s_k \mathbf{x}_i^{\text{ap}}
$$
\n(15)

where two frequencies $\omega_i/(2\pi)$ were introduced (1 and 2 cycles per year). At a specific frequency, the signal is parameterized by sine and cosine amplitudes making the model linear. The term δPSD(t) has been estimated beforehand from GNSS input coordinate series, the best model being selected by a statistical method. Input series have then been corrected for that model which was assumed identical for colocated stations.

Even if the post-seismic models δ PSD(t) were set up and the associated parameters were estimated in the stacking, the addition of these parameters would not cause an additional rank deficiency in the normal equation of the stacking. Indeed, post-seismic models only affect a part of the whole station network. Conversely, as periodic parameters are added for all stations, supplementary rank deficiencies are added to the normal equation for each frequency (Petrov and Ma, 2003) due to the simultaneous estimation of transformation parameters. For example, any periodic signal (a^j, b^j) added to all stations can be canceled by subtracting the same signal from the estimated translations.. As a consequence, condition equations must be added for every frequency. It is either possible to add constraints on the station periodic signals or on the transformation parameters.

As done for equation (7), it is possible to first cancel periodic signals at every frequency $\omega_i/(2\pi)$ in the transformation parameters, which gives for a transformation q^2 parameter p :

$$
(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{p} = 0 \quad ; \quad \mathbf{A} = \begin{pmatrix} \cos\omega_j(t_1 - t_0) & \sin\omega_j(t_1 - t_0) \\ \vdots & \vdots \\ \cos\omega_j(t_K - t_0) & \sin\omega_j(t_K - t_0) \end{pmatrix} \quad ; \quad \mathbf{p} = \begin{pmatrix} p_1 \\ \vdots \\ p_K \end{pmatrix} \quad (16)
$$

This condition can be used for canceling the annual signal in the scale parameter. Indeed, it is likely that most of the seasonal variations in the scale parameters be related to loading signals (Collilieux et al., 2010). If applied to the translation parameters, this constraint transfers any seasonal net-translation of the network to station seasonal parameters.

The alternative approach consists in constraining the seasonal parameters a_i^j and **b**_iⁱ. By writing $\mathbf{\alpha}^j = [\mathbf{a}_1^j \dots \mathbf{a}_i^j \dots \mathbf{a}_n^j]^T$ and $\mathbf{\beta}^j = [\mathbf{b}_1^j \dots \mathbf{b}_i^j \dots \mathbf{b}_n^j]^T$, minimum constraints can be added as follows:

$$
(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\boldsymbol{\alpha}^{\mathrm{j}}-\boldsymbol{\alpha}^{\mathrm{j},\mathrm{ap}})=(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\boldsymbol{\alpha}_{\mathrm{ex}}^{\mathrm{i}}-\boldsymbol{\alpha}^{\mathrm{j},\mathrm{ap}})
$$
(17)

$$
(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\boldsymbol{\beta}^{\mathrm{j}}-\boldsymbol{\beta}^{\mathrm{j},\mathrm{ap}})=(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}(\boldsymbol{\beta}_{\mathrm{ex}}^{\mathrm{i}}-\boldsymbol{\beta}^{\mathrm{j},\mathrm{ap}})
$$

where α_{ex}^{j} and β_{ex}^{j} have been fitted from an existing loading model or from GRACE observations.

This algebraic constraint provides an alternative to the equations derived by Dermanis (2008) to explicitly cancel the relative angular momentum of the network accounting for seasonal signals. In order to be relevant, the constraint of equation (17) should be applied with respect to a model which verifies a no-net-rotation condition. In the loading theory that has been briefly introduced in section III.1, a rotation is described by the toroidal term of degree-1. But no degree-1 toroidal deformation is possible when the external forcing is a load on the spherical surface (Métivier et al., 2006). Thus, the no-net-rotation condition of the loading model is transferred to the estimated seasonal terms as currently done for the orientation linear time evolution of the reference frame.

III.3 Estimated seasonal parameters

The estimated annual and semi-annual periodic signals as the result of the stacking are dependent on the constraints that have been applied as described in the previous section. In ITRF2014, the following options have been used in the stacking of the four techniques:

- Equation (16) was applied to cancel annual and semi-annual signals in the translation and scale parameters.
- Equation (17) was applied to define the orientation of the seasonal parameters over well distributed sets of stations. No external loading model was used as reference for the constraints, i.e. α_{ex}^{j} and β_{ex}^{j} were set to zero.

The first constraint impacts the origin of seasonal periodic signals which is theoretically CM for satellite techniques. However, as already found in various studies, the origin of DORIS, SLR and GNSS frames differ at seasonal time-scale (see figure 9 from Rebischung et al., 2016; Altamimi et al., 2016; Ray et al., 1999). In addition, for ITRF2004, geocenter parameters have been introduced in all input GNSS frames while GNSS station coordinates have been aligned to IGb08 frame (Rebischung et al., 2016) which makes the estimated ITRF2014 GNSS seasonal parameters more consistent with CF frame. The obtained frame origin of VLBI seasonal signals is also arbitrary because the origins of the input VLBI coordinate datasets have been constrained to an a priori coordinate frame beforehand. It is thus likely that mean seasonal biases exist between technique-specific estimated seasonal signals.

The second constraint affects the estimated polar motion coordinates. Due to the limited number of stations and their distribution in space, the station displacements due to loading generate a small residual rotation. According to Collilieux et al. (2012), this residual rotation can be mitigated when a well-distributed network of station is used, which explains the constraint chosen to define the orientation of the seasonal signals in the ITRF2014 computation.

Because the seasonal signals of the different techniques are not provided in the same reference frame as discussed above and because they were not combined in ITRF2014, they have not been provided to enhance the ITRF kinematic coordinate model. The question of delivering a unique seasonal displacement for all stations at the same co-location site needs also to be addressed. Indeed, whereas we could in principle expect the averaged seasonal signal to be more reliable than any individual one at co-location sites, it cannot be ensured as long as the origin of their discrepancies is not understood. In addition, seasonal signals should be provided in the CM frame according to the ITRS specifications. It would require that the SLR origin of the seasonal parameters be transferred to other technique seasonal parameters. And this could be done reliably only if technique specific seasonal parameters are similar (in the sense of a 6-parameter similarity) over the set of common stations. As a conclusion, seasonal parameters at co-located stations should be carefully analyzed before attempting their combination.

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